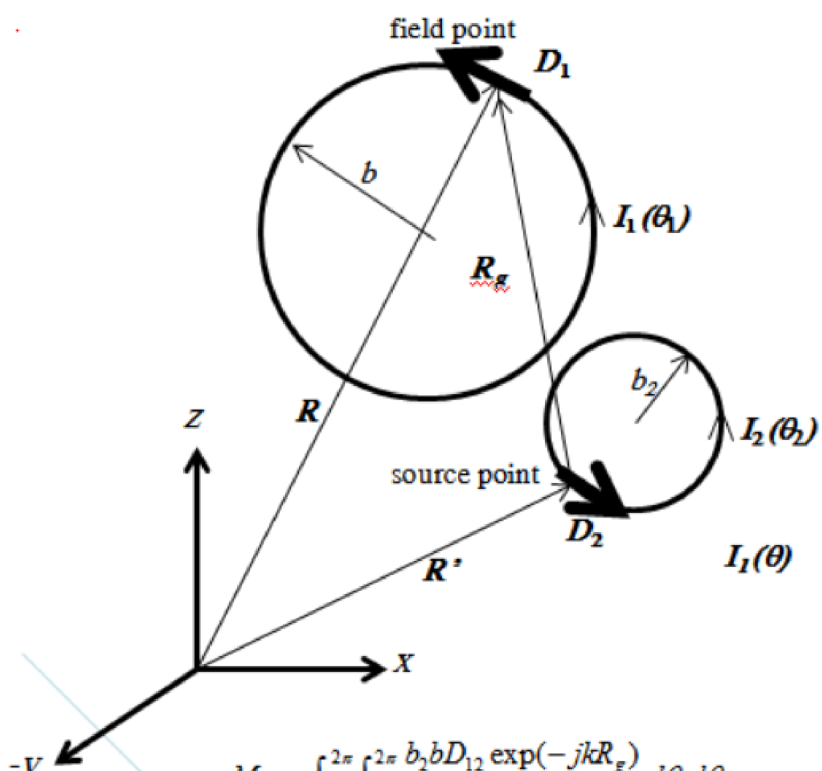
Small Loop Mutual Coupling Analysis

Kai Siwiak, KE4PT

20 July 2015 loop coupled / to second loop

This is an approximate for the Mutual Coupling to ground using the Neumann formula to estimate the mutual inductance and hence the mutual impedance between two loops; assumes UNIFORM current on the conductor.



$$M_{12} = \int_0^{2\pi}\int_0^{2\pi} \frac{b_2 b D_{12} \exp(-jkR_g)}{R_g}\, d\theta_1 d\theta_2$$

$$D_{12} = \cos(\theta_1)\cos(\theta_2) + \sin(\theta_1)\sin(\theta_2)$$

$$R_g = \sqrt{\begin{array}{l}\left[b_2 \sin\theta_2 - b\sin\theta_1 + X\right]^2 + \cdots \\ + \left[b_2\cos\theta_2 - b\cos\theta_1 + Z\right]^2 + Y^2\end{array}}$$

LOOPS ARE IN THE ZX plane

Kai Siwiak
20 July 2013

Freq, MHz: $f := 14.1$

$\mu := 4\cdot\pi\cdot10^{-7}$ $c := 299792456$ $\varepsilon := \dfrac{1}{\mu\cdot c^2}$ $\eta := c\cdot\mu$ $\omega := 2\cdot\pi\cdot f\cdot10^{6}$ $k := \omega\cdot\sqrt{\mu\cdot\varepsilon}$

Loop radius, m $b := 0.45339$ m $k\cdot b = 0.134$

Wire radius, m $a := 0.004064$

Second smaller loop: $b2 := 0.077$ m $a2 := 0.002$ $b2 = 0.077$

Loops in the same plane, but centers displaced by w from each othe $w := b - b2 - 5.5\cdot(a + a2)$
same as in 4nec2 model. $w = 0.34304$

(Z, including dipole higer order terms for, (kb)<0.4:)

$$ZLo(k,b,a) := \eta\cdot\frac{\pi}{6}\cdot(k\cdot b)^4\cdot\left[1-\left(\frac{a}{b}\right)^2\right]\cdot\left[1+8\cdot(k\cdot b)^2\right]+j\cdot\eta\cdot k\cdot b\cdot\left[\ln\left(8\cdot\frac{b}{a}\right)-2+\frac{2}{3}\cdot(k\cdot b)^2\right]\cdot\left[1+8\cdot(k\cdot b)^2\right]$$

Include the loop losses $Rloss := .26187 - 0.073$ at 14.1 MHz

$ZLo(k,b,a) = 0.073 + 277.4$ $ZL(k,b,a) := ZLo(k,b,a) + Rloss$ $ZL(k,b,a) = 0.262 + 277.423i$

Loop of radius 'b' in ZX plane, centers displace by (X,Y,Z) $X := 0$ $Y := a$ $Z := w$

$$L1dotL2(\theta1,\theta2) := (\cos(\theta1)\cdot\cos(\theta2) + \sin(\theta1)\cdot\sin(\theta2))$$

$$Rg(\theta1,\theta2,X,Y,Z,b,b2) := \sqrt{\left[(b2\cdot\sin(\theta2)-b\cdot\sin(\theta1))+X\right]^2+\left[(b2\cdot\cos(\theta2)-b\cdot\cos(\theta1))+Z\right]^2+Y^2}$$

$$M12g(X,Y,Z,b,b2) := \frac{\mu}{4\cdot\pi}\cdot\int_0^{2\cdot\pi}\int_0^{2\cdot\pi}\frac{(b\cdot b2)\cdot L1dotL2(\theta1,\theta2)}{Rg(\theta1,\theta2,X,Y,Z,b,b2)}\,d\theta2\,d\theta1$$ $M12g(X,Y,Z,b,b2) = 5.7114\times10^{-8}$

$Lloop := \dfrac{\eta\cdot k\cdot b}{\omega}\cdot\left(\ln\left(8\cdot\dfrac{b}{a}\right)-2\right)$ $Lloop := \mu\cdot b\cdot\left(\ln\left(8\cdot\dfrac{b}{a}\right)-2\right)$

$M12g(0,a,0,b,b) = 2.73144\times10^{-6}$ self term is okay $Lloop = 2.73138\times10^{-6}$

$M12g(0,a,0,b2,b) = 2.60941\times10^{-8}$ mutual term, loops centered

$M12g(0,a,w,b2,b) = 5.7114\times10^{-8}$ mutual term, loops displaced by w in ZX plane; use HF term below

$b = 0.453$ $b2 = 0.077$ Loop centers displaced by "w"

$X = 0$ $Y = 4.064 \times 10^{-3}$ $Z = 0.343$ $w = 0.343$

$a = 4.064 \times 10^{-3}$

Mutual coupling inductance: $M12g(X,Y,Z,b,b2) = 5.7114 \times 10^{-8}$ Henry

$ZL(k,b,a) = 0.262 + 277.423i$

$Z12g(X,Y,Z,b,b2) := j \cdot \omega \cdot M12g(X,Y,Z,b,b2)$ $Z11g(a,b) := j \cdot \omega \cdot M12g(0,a,0,b,b)$ $X = 0$

$Z12g(X,Y,Z,b,b2) = 5.06i$ $Z11g(a,b) = 241.986i$ $Z = 0.343$ $Y = 4.064 \times 10^{-3}$

Change in loop inductance due to M12 term: $InductanceChange := \frac{M12g(X,Y,Z,b,b2) + M12g(0,a,0,b,b)}{M12g(0,a,0,b,b)}$

$InductanceChange = 1.02091$

self inductance of main term: $M12g(0,a,0,b,b) = 2.73144 \times 10^{-6}$

$Lloop = 2.73138 \times 10^{-6}$

Effect on loop reonance, at 14.1 MHz:

$$dFres := \left(\frac{M12g(0,a,0,b,b)}{M12g(X,Y,Z,b,b2) + M12g(0,a,0,b,b)}\right)$$

$dFres = 0.98$

Resonating capacitance: $C := \frac{1}{\omega \cdot Im(ZL(k,b,a))}$ $C \cdot 10^{12} = 40.687$ pF

$Cmut := \frac{dFres}{\omega \cdot Im(ZL(k,b,a))}$ $Cmut \cdot 10^{12} = 39.854$ pF

mutual coupling increases inductance, so resonating cap needs to be reduced by C-Cmut!

$(C - Cmut) \cdot 10^{12} = 0.833$ pF

For High Frequency formula, see Jordan and Balmain, Electromagnetic Waves and Radiating Systems, Sec. Ed. eqn (14-146).

$$\mathrm{M12gHF}(X,Y,Z,b,b2) := \frac{\mu}{4\cdot\pi}\cdot\int_0^{2\cdot\pi}\int_0^{2\cdot\pi}\frac{(b\cdot b2)\cdot \mathrm{L1dotL2}(\theta 1,\theta 2)\,\exp(-j\cdot k\cdot \mathrm{Rg}(\theta 1,\theta 2,X,Y,Z,b,b2))}{\mathrm{Rg}(\theta 1,\theta 2,X,Y,Z,b,b2)}\,d\theta 2\,d\theta 1$$

Add this term as M12 to the loop impedance: $\mathrm{M12gHF}(X,Y,Z,b,b2) = 5.73045\times 10^{-8} - 2.06145i\times 10^{-11}$

Compared to using tyhe low frequency formula: $\mathrm{M12g}(X,Y,Z,b,b2) = 5.7114\times 10^{-8}$

HF correction factor: $\left|\frac{\mathrm{M12gHF}(X,Y,Z,b,b2)}{\mathrm{M12g}(X,Y,Z,b,b2)}\right| = 1.003$

Comparison of loow freq and HF formulas

$\mathrm{M12g}(0,a,0,b,b) = 2.73144\times 10^{-6}$ $\mathrm{M12gHF}(0,a,0,b,b) = 2.7382\times 10^{-6} - 7.14943i\times 10^{-10}$

$\mathrm{M12g}(0,a,0,b2,b) = 2.60941\times 10^{-8}$ $\mathrm{M12gHF}(0,a,0,b2,b) = 2.63239\times 10^{-8} - 2.0657i\times 10^{-11}$

$\mathrm{M12g}(0,a,w,b2,b) = 5.7114\times 10^{-8}$ $\mathrm{M12gHF}(0,a,w,b2,b) = 5.73045\times 10^{-8} - 2.06145i\times 10^{-11}$

Sanity check: for constant current:

$$\mathrm{ZLcons}(k,b,a) := \eta\cdot\frac{\pi}{6}\cdot(k\cdot b)^4 + j\cdot\eta\cdot k\cdot b\cdot\left(\ln\left(8\cdot\frac{b}{a}\right) - 2\right)$$

$$\mathrm{ZLcons}(k,b,a) = 0.06357 + 241.98097i$$

$$j\cdot\omega\cdot\mathrm{M12gHF}(0,a,0,b,b) = 0.06334 + 242.58553i$$

The HF formula gives the correct radiation resistance as well as the correct reactance, for constant current, to within 0.25%.

$$\left(\left|\frac{j\cdot\omega\cdot\mathrm{M12gHF}(0,a,0,b,b)}{\mathrm{ZLcons}(k,b,a)}\right| - 1\right)\cdot 100 = 0.24984 \qquad \%$$