Small Loop/Dipole Mutual Coupling To Ground Analysis

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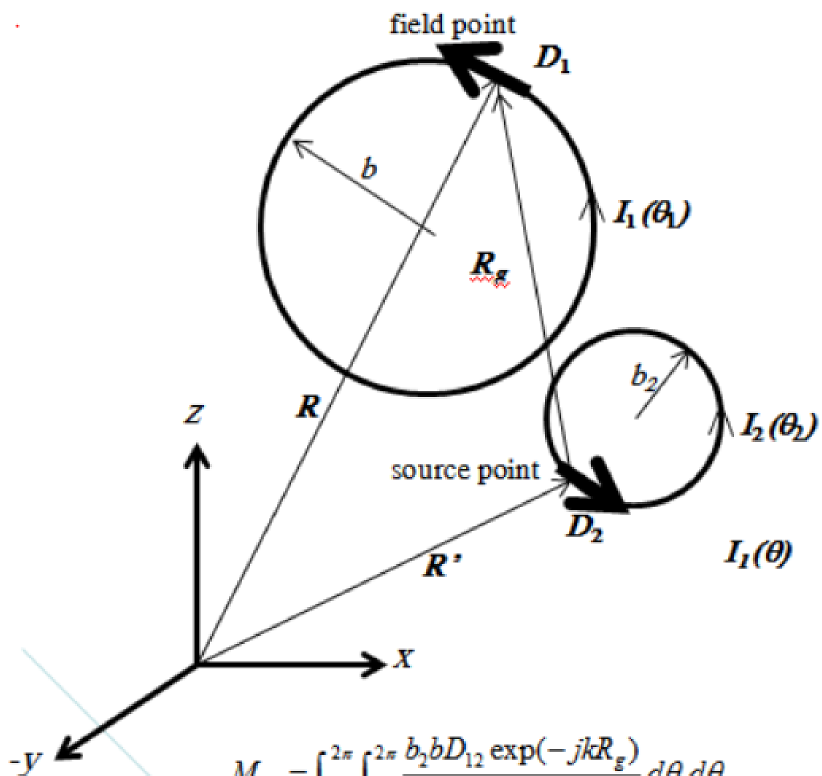
4 July 2013

This is an approximate Mutual Coupling to ground using the Neumann integral to estimate the mutual inductance and hence the mutual impedance between the loop and its image; assumes UNIFORM current on the conductor.



$$M_{12} = \int_0^{2\pi}\int_0^{2\pi} \frac{b_2 b D_{12}\exp(-jkR_g)}{R_g}\, d\theta_1 d\theta_2$$

$$D_{12} = \cos(\theta_1)\cos(\theta_2) + \sin(\theta_1)\sin(\theta_2)$$

$$R_g = \sqrt{\begin{matrix}[b_2\sin\theta_2 - b\sin\theta_1 + X]^2 + \cdots \\ +[b_2\cos\theta_2 - b\cos\theta_1 + Z]^2 + Y^2\end{matrix}}$$

Loops of radius 'b' in ZX plane, centers displace by (X,Y,Z)

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20 July 2013

Freq, MHz: $f := 14.1$

$\mu := 4\cdot\pi\cdot 10^{-7}$ $c := 299792456$

$\varepsilon := \dfrac{1}{\mu\cdot c^2}$ $\eta := c\cdot\mu$ $\omega := 2\cdot\pi\cdot f\cdot 10^6$ $k := \omega\cdot\sqrt{\mu\cdot\varepsilon}$

Loop radius, m $b := 0.45339$ $k\cdot b = 0.134$ $b = 0.453$

Wire radius, m $a := 0.004064$ $a = 4.064\times 10^{-3}$

Height above ground, m $h := b + 0.1$

(Z, including dipole mode terms for, (kb)<0.4:)

$$Z(k,b,a) := \eta\cdot\frac{\pi}{6}\cdot(k\cdot b)^4\cdot\left[1-\left(\frac{a}{b}\right)^2\right]\cdot\left[1+8\cdot(k\cdot b)^2\right] + j\cdot\eta\cdot k\cdot b\cdot\left[\ln\left(8\cdot\frac{b}{a}\right) - 2 + \frac{2}{3}\cdot(k\cdot b)^2\right]\cdot\left[1+8\cdot(k\cdot b)^2\right]$$

include mutual coupling to ground h meter below, reflection coefficient of ground. $Gamma := 1$
For vertical (Y or X) separation ground image is 2X or 2Y away!!:

Loops of radius 'b' in ZX plane, centers displace by (X,Y,Z)

For High Frequency version, see Jordan and Balmain, Electromagnetic Waves and Radiating Systems, Sec. Ed. eqn (14-146).

$$L1dotL2(\theta1,\theta2) := b^2\cdot(\cos(\theta1)\cdot\cos(\theta2) + \sin(\theta1)\cdot\sin(\theta2))$$

$$Rg(\theta1,\theta2,X,Y,Z,b) := \sqrt{[b\cdot(\sin(\theta2)-\sin(\theta1)) + X]^2 + [b\cdot(\cos(\theta2)-\cos(\theta1)) + Z]^2 + Y^2}$$

$$M12gHF(X,Y,Z,b) := \frac{\mu}{4\cdot\pi}\cdot\int_0^{2\cdot\pi}\int_0^{2\cdot\pi}\frac{L1dotL2(\theta1,\theta2)\exp(-j\cdot k\cdot Rg(\theta1,\theta2,X,Y,Z,b))}{Rg(\theta1,\theta2,X,Y,Z,b)}\,d\theta2\,d\theta1$$

$$M12g(X,Y,Z,b) := \frac{\mu\cdot b^2}{4\cdot\pi}\cdot\int_0^{2\cdot\pi}\int_0^{2\cdot\pi}\frac{(\cos(\theta1)\cdot\cos(\theta2)+\sin(\theta1)\cdot\sin(\theta2))}{Rg(\theta1,\theta2,X,Y,Z,b)}\,d\theta2\,d\theta1$$

$$Z12g(X,Y,Z,b) := j\cdot\omega\cdot M12g(X,Y,Z,b)$$

Radiated power Rrad is the imaginary part of I^2 M12gHF, so radiation resistance is :

$$\mathrm{Rrad}(X,Y,Z,b) := \frac{-(\mu\cdot\omega)}{4\cdot\pi}\cdot\int_0^{2\cdot\pi}\int_0^{2\cdot\pi}\frac{\mathrm{L1dotL2}(\theta1,\theta2)\sin(-k\cdot\mathrm{Rg}(\theta1,\theta2,X,Y,Z,b))}{\mathrm{Rg}(\theta1,\theta2,X,Y,Z,b)}\,d\theta2\,d\theta1$$

$\mathrm{Rrad}(0,0,a,b) = 0.063$

$\eta\cdot\frac{\pi}{6}\cdot(k\cdot b)^4 = 0.064$ for uniform current, no higher order modes

SANITY CHECK bring loops to within 'a' of each other along Z and compare to self-inductance:

Loop inductance: $M(k,b,a) := \frac{\eta\cdot k\cdot b\cdot\left(\ln\left(8\cdot\frac{b}{a}\right)-2\right)}{\omega}$ $M11(b,a) := \mu\cdot b\cdot\left(\ln\left(8\cdot\frac{b}{a}\right)-2\right)$

Self inductance at 'a' distance: $M(k,b,a) = 2.731\times10^{-6}$ $M11(b,a) = 2.731\times10^{-6}$

$M12g(0,a,0,b) = 2.731\times10^{-6}$

For the vertcal loop h/b loop radii over ground, ant height

$$\mathrm{Zadditional}(h,b) := \frac{-\left(Z12g(0,2\cdot h,0,b)^2\right)}{Z(k,b,a)}$$

$\frac{h}{b} = 1.221$

$\mathrm{Zadditional}(h,b) = 1.251\times10^{-5} - 0.048i$

$f = 14.1$ $Z12g(0,2\cdot h,0,b) = 3.64i$ $Z(k,b,a) = 0.073 + 277.423i$

$b = 0.453$

$a = 4.064\times10^{-3}$ $\mathrm{Ztot}(k,b,a) := Z(k,b,a) - \mathrm{Zadditional}(h,b)$ $\mathrm{Ztot}(k,b,a) = 0.073 + 277.471i$

$h = 0.553$

Capacitance, pF, to resonate isolated loop:

$\left(\omega\cdot\left|\mathrm{Im}(Z(k,b,a))\right|\right)^{-1}\cdot10^{12} = 40.687$

Capacitance, pF, to resonate coupled loop:

$\left(\omega\cdot\left|\mathrm{Im}(\mathrm{Ztot}(k,b,a))\right|\right)^{-1}\cdot10^{12} = 40.68$

$HH := 51$ $\quad dmin := b + a$ $\quad dmax := 3.98 \cdot b$ $\quad b = 0.453$

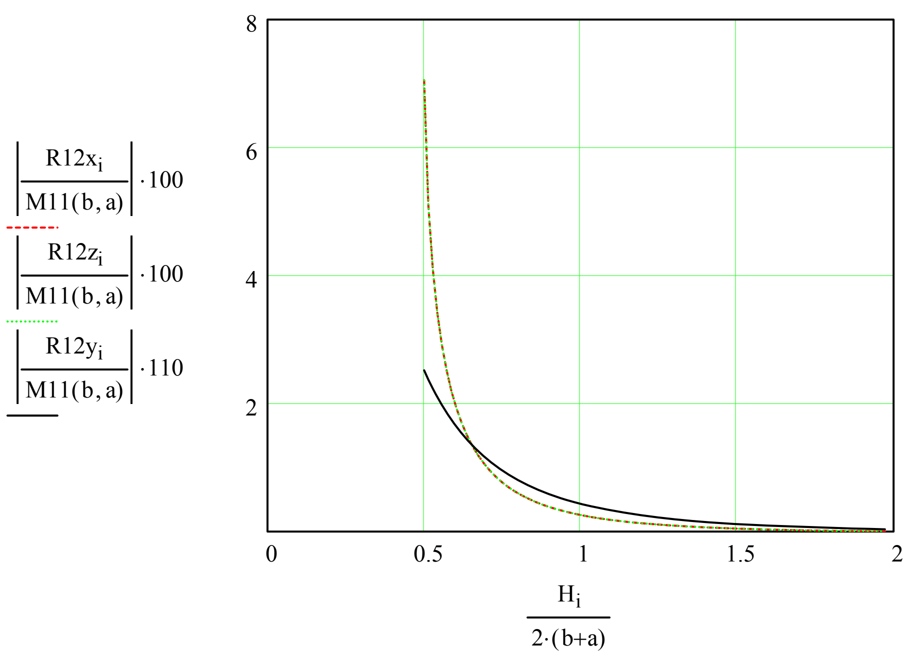
$i := 0 .. HH$ $\quad H_i := dmin \cdot \exp\left(\frac{i}{HH} \cdot \ln\left(\frac{dmax}{dmin}\right)\right)$ $\quad H_{HH} = 1.804$

$R12z_i := M12g(0, 0, 2 \cdot H_i, b)$ $\quad R12y_i := M12g(0, H_i \cdot 2, 0, b)$ $\quad M12g(0, a, 0, b) = 2.731 \times 10^{-6}$

$R12x_i := M12g(2 \cdot H_i, 0, 0, b)$ $\quad M11(b, a) = 2.731 \times 10^{-6}$

Loops of radius 'b' in ZX plane, centers displace by (X,Y,Z) $M12g(X, Y, Z, b)$ $\quad b = 0.453$

R12z is mutual inductance with loops in ZX plane, centers displaced in Z (edge cloupled)
R12x is mutual inductance with loops in ZX plane, centers displaced in X (edge coupled)
R12y is mutual inductance with loops in ZX plane, centers displaced in Y (parallel-coupled)



M11(b,a) is the self inductance.

Percent field coupling to the ground, vs. center of loop height (in loop diameters) above ground

loop, outer diameter 2(b+a), resting on ground

loop center one diameter above the ground

$$\frac{M12g[0, 2 \cdot (b + a), 0, b]}{M11(b, a)} \cdot 100 = 2.31$$

Good and Poor ground:

$$\frac{M12g[0,2\cdot(2\cdot b),0,b]}{M11(b,a)}\cdot 100 = 0.432$$

$$\varepsilon t(f,\varepsilon r,\sigma) := \varepsilon r + j\cdot\frac{\sigma}{2\cdot\pi\cdot\varepsilon\cdot f\cdot 10^{6}}$$

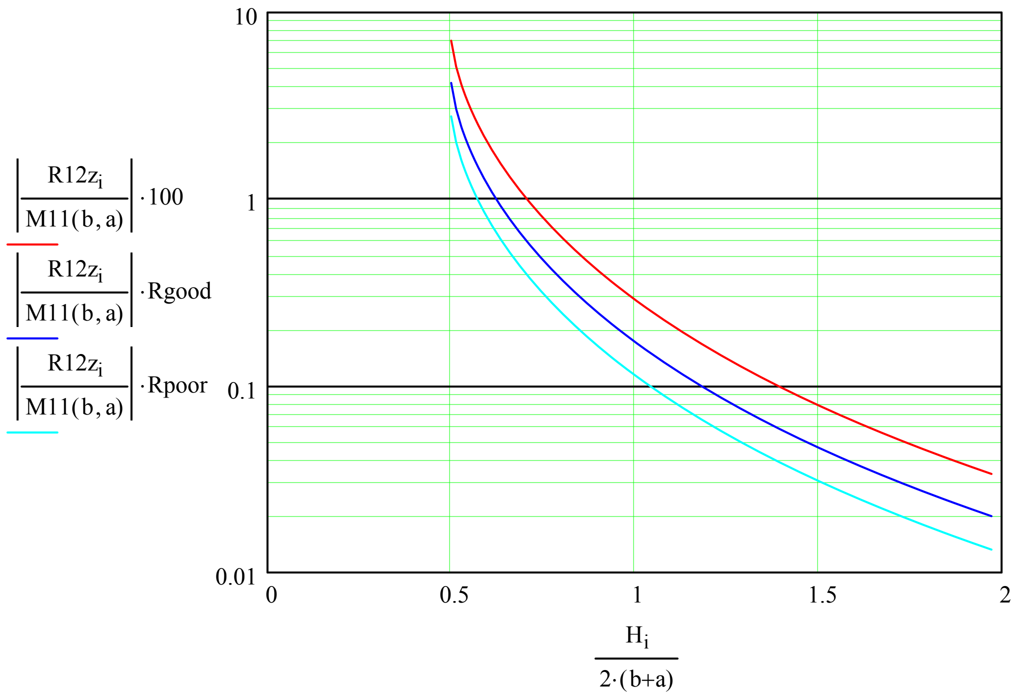
$$\varepsilon t(14.2,13,0.005) = 13 + 6.329i$$

$$\mathrm{Refl}(f,\varepsilon,\sigma) := \left|\frac{\sqrt{\varepsilon t(f,\varepsilon,\sigma)}-1}{\sqrt{\varepsilon t(f,\varepsilon,\sigma)}+1}\right|$$

$Rgood := 100\cdot \mathrm{Refl}(14.1,13,0.005)$ $Rgood = 59.357$ % reflection

$Rpoor := 100\cdot \mathrm{Refl}(14.2,5,0.001)$ $Rpoor = 39.339$ % reflection

%coupling; 100*M12/M11



Center of loop height above ground in loop diameters.