TRANSMISSION LINE PROPERTIES FROM MEASURED DATA

Version 8.1.01 **July 5, 1999**

Impedance is measured with cable terminated with open and short circuits at far end.
See "Transmission Line Properties from Measured Data" by Frank Witt, AI1H
ARRL Antenna Compendium, Volume 6

PHYSICAL CONSTANTS AND CONVERSION FACTORS

$c_{Metric} := 299.792458$ Megameters/sec

$k_1 := .3048$ meters/foot

$c_{English} := \frac{c_{Metric}}{k_1}$

$c_{English} = 983.571056$ Megafeet/sec

$k_2 := \frac{\ln(10)}{20}$

$k_2 = 0.1151292546$ nepers/dB

$k_3 := \frac{100\cdot\pi}{c_{English}\cdot k_2}$

$k_3 = 2.774332$

MEASURED DATA AND ESTIMATE OF VELOCITY FACTOR AND INSULATION EXPONENT

Replace these data with measured data for other cable segments. Some examples are in TLMEASDATAV8.MCD.

22 ft 3½ in RG58C cable
(Wireman 127)
measured at 3.6 MHz
by Pete Schuch, WB2UAQ
with HP 8753B
Network Analyzer

$F := 3.6$ MHz

$Z_{OCF} := .80 - 50.20\cdot j$ ohms

$Z_{SCF} := 3.53 + 51.78\cdot j$ ohms

$Length := 22.29$ feet

$VF_{est} := .66$

$g := 1.0$

BASIC CABLE PROPERTIES AT THE TEST FREQUENCY

Complex characteristic impedance at test frequency, F

$Z_{0F} := \sqrt{Z_{OCF}\cdot Z_{SCF}}$

$Z_{0F} = 51.029 - 1.33j$ ohms

$R_{0F} := \mathrm{Re}(Z_{0F})$

$X_{0F} := \mathrm{Im}(Z_{0F})$

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Propagation constant at the test frequency, F

$$\gamma'_F := \frac{\operatorname{atanh}\left(\sqrt{\frac{Z_{SCF}}{Z_{OCF}}}\right)}{Length} \qquad \alpha_F := \operatorname{Re}(\gamma'_F) \text{ nepers/foot} \qquad \beta'_F := \operatorname{Im}(\gamma'_F) \text{ radians/foot}$$

There is an ambiguity in the calculation of β, the phase constant. This is resolved by using an estimate of the velocity factor to calculate an estimate of β. Then n, the estimated nearest number of half wavelengths which make up the cable length, is calculated.

$$\beta_{Fest} := \frac{2\cdot\pi\cdot F}{VF_{est}\cdot c_{English}} \qquad n_{Fest} := \frac{Length}{\pi}\cdot\left(\beta_{Fest} - \beta'_F\right) \qquad n_{Fest} = -5.42\cdot 10^{-3}$$

$$\operatorname{mantissa}(n_{Fest}) := n_{Fest} - \operatorname{floor}(n_{Fest}) \qquad n_F := \operatorname{if}\left(\operatorname{mantissa}(n_{Fest}) < .5, \operatorname{floor}(n_{Fest}), \operatorname{ceil}(n_{Fest})\right)$$

$$\beta_F := \operatorname{Im}(\gamma'_F) + \frac{n_F\cdot\pi}{Length} \qquad \gamma_F := \alpha_F + \beta_F\cdot j$$

$\alpha_F = 9.423\cdot 10^{-4}$ nepers/foot $\qquad \beta_F = 0.036$ radians/foot $\qquad n_F = 0$

Velocity factor and effective dielectric constant at frequency F

$$VF := \frac{2\cdot\pi\cdot F}{c_{English}\cdot\beta_F} \qquad \varepsilon_{eff} := \frac{1}{VF^2}$$

$VF = 0.646 \qquad \varepsilon_{eff} = 2.397$

Resistance, inductance, conductance and capacitance per foot at frequency F

$R_F := \operatorname{Re}(\gamma_F\cdot Z_{0F}) \qquad R_F = 0.0955$ ohms/foot

$L_F := \dfrac{\operatorname{Im}(\gamma_F\cdot Z_{0F})}{2\cdot\pi\cdot F} \qquad L_F = 0.0803$ µH/foot

$G_F := \operatorname{Re}\left(\dfrac{\gamma_F}{Z_{0F}}\right) \qquad G_F = 2.727\cdot 10^{-7}$ siemens/foot

$C_F := \dfrac{\operatorname{Im}\left(\frac{\gamma_F}{Z_{0F}}\right)}{2\cdot\pi\cdot F\cdot 10^{-6}} \qquad C_F = 30.85$ pF/foot

$$Z_{0RLGCF} := \sqrt{\frac{R_F + 2\cdot\pi\cdot F\cdot L_F\cdot j}{G_F + 2\cdot\pi\cdot F\cdot 10^{-6}\cdot C_F\cdot j}}$$

$Z_{0RLGCF} = 51.028914 - 1.330442j$ ohms

Compare with:

$Z_{0F} = 51.028914 - 1.330442j$ ohms

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FREQUENCY DEPENDENCE

Characteristic impedance and propagation constant as a function of frequency. The insulation exponent, g, is a measure of the rate at which the conductance per foot increases with frequency.

$$R(f) := R_F \cdot \sqrt{\frac{f}{F}} \qquad G(f) := G_F \cdot \left(\frac{f}{F}\right)^g \qquad L(f) := L_F \qquad C(f) := C_F$$

$$Z_0(f) := \sqrt{\frac{R(f) + 2\cdot\pi\cdot f\cdot L(f)\cdot j}{G(f) + 2\cdot\pi\cdot f\cdot 10^{-6}\cdot C(f)\cdot j}} \qquad R_0(f) := \mathrm{Re}\left(Z_0(f)\right) \qquad X_0(f) := \mathrm{Im}\left(Z_0(f)\right)$$

$$\alpha(f) := \frac{R(f)}{2\cdot R_0(f)} + \frac{G(f)\cdot\left(\left|Z_0(f)\right|\right)^2}{2\cdot R_0(f)} \qquad \beta(f) := \beta_F\cdot\frac{f}{F} \qquad \gamma(f) := \alpha(f) + \beta(f)\cdot j$$

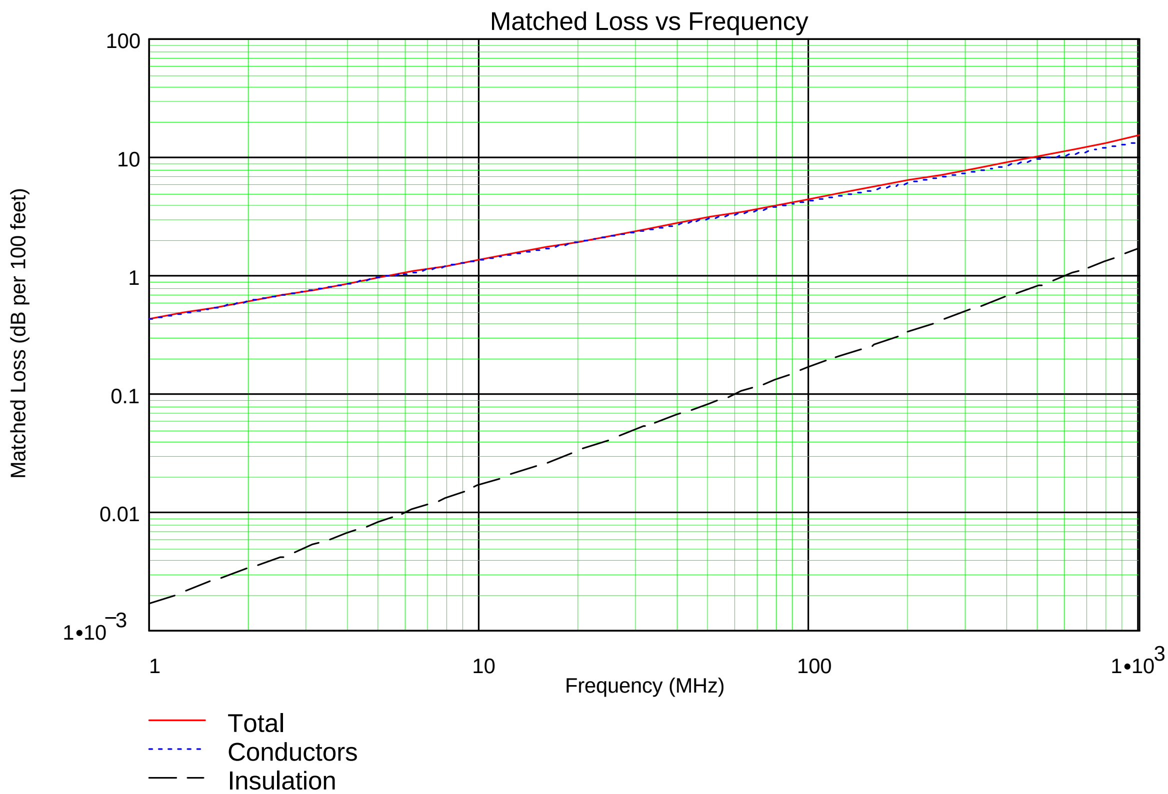
Matched loss per 100 feet

$$A_0(f) := \frac{\alpha(f)}{k_2}\cdot 100 \qquad \alpha_{conductors}(f) := \frac{R(f)}{2\cdot R_0(f)} \qquad \alpha_{insulation}(f) := \frac{G(f)\cdot\left(\left|Z_0(f)\right|\right)^2}{2\cdot R_0(f)}$$

$$A_{0conductors}(f) := \frac{\alpha_{conductors}(f)}{k_2}\cdot 100 \qquad A_{0insulation}(f) := \frac{\alpha_{insulation}(f)}{k_2}\cdot 100$$

$$j := 0..30 \qquad f_j := 10^{\frac{j}{10}}$$

See next page for graph.



Crossover frequency. This is the frequency where the loss due to the conductors is equal to the loss due the insulation. At this frequency the characteristic impedance is real. Two iterations are used.

First iteration:

$$F_X := \mathrm{if}\left[g = 0.5, 0, F\cdot\left(\frac{R_F}{G_F\cdot R_{0F}^{\;2}}\right)^{\frac{2}{2\cdot g-1}}\right]$$

$F_X = 65078$ MHz

$Z_0(F_X) = 51.011047 - 6.982415j\cdot 10^{-6}$ ohms

Note that $\mathrm{Im}(Z_0)$ is near zero

Second iteration:

$$F_X := \mathrm{if}\left[g = 0.5, 0, F_X\cdot\left(\frac{R(F_X)}{G(F_X)\cdot R_0(F_X)^2}\right)^{\frac{2}{2\cdot g-1}}\right]$$

$F_X = 65170$ MHz

$Z_0(F_X) = 51.011047$ ohms

Note that $\mathrm{Im}(Z_0) = 0$

Compare with:

$\sqrt{\dfrac{L(F_X)}{C(F_X)\cdot 10^{-6}}} = 51.011047$ ohms

$\sqrt{\dfrac{R(F_X)}{G(F_X)}} = 51.011047$ ohms

SOLUTIONS FOR AN ARBITRARY FREQUENCY AND LENGTH

New frequency and length. Replace these with the desired frequency and length.

$F_{NEW} := 28.8\ \mathrm{MHz}$ $\quad$ $Length_{NEW} := 50\ \mathrm{feet}$

Electrical angle and length in wavelengths at new frequency and length

$\theta(f) := \beta(f) \cdot Length_{NEW} \cdot \frac{180}{\pi}$ $\quad$ $\theta(F_{NEW}) = 816.07$ degrees

$Wavelengths(f) := \frac{Length_{NEW} \cdot f}{c_{English} \cdot VF}$ $\quad$ $Wavelengths(F_{NEW}) = 2.267$

Interesting lengths - Cable lengths required to create half, quarter and eighth wavelength at new frequency

$Length_{half}(f) := \frac{c_{English} \cdot VF}{2 \cdot f}$ $\quad$ $Length_{half}(F_{NEW}) = 11.03$ feet

$Length_{quarter}(f) := \frac{Length_{half}(f)}{2}$ $\quad$ $Length_{quarter}(F_{NEW}) = 5.51$ feet

$Length_{eighth}(f) := \frac{Length_{half}(f)}{4}$ $\quad$ $Length_{eighth}(F_{NEW}) = 2.76$ feet

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Interesting frequencies - Frequencies at which the new cable segment is a half, quarter and eighth wavelength.

$F_{half} := \frac{c_{English} \cdot VF}{2 \cdot Length_{NEW}}$ $\quad$ $F_{quarter} := \frac{F_{half}}{2}$ $\quad$ $F_{eighth} := \frac{F_{half}}{4}$

$F_{half} = 6.352$ MHz $\quad$ $F_{quarter} = 3.176$ MHz $\quad$ $F_{eighth} = 1.588$ MHz

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Characteristic impedance at the new frequency

$Z_0(F_{NEW}) = 51.013 - 0.464j$ ohms

"Real" characteristic impedance at the new frequency

Physical characteristic impedance

$$Z_{0LC}(f) := \sqrt{\frac{L(f)}{C(f)} \cdot 10^6}$$

$Z_{0LC}(F_{NEW}) = 51.011$ ohms

Compare with:

$R_0(F_{NEW}) = 51.013$ ohms

$|Z_0(F_{NEW})| = 51.015$ ohms

Define: $Z_{0real} := Z_{0LC}(F)$

Estimate of "real" characteristic impedance at the new frequency from C and VF.

C may be measured at low frequencies with a capacitance meter. The velocity factor may be found using cable segments with short and/or open circuit terminations.

$$Z_{0LCest}(f) := \frac{10^6}{c_{English} \cdot C(f) \cdot VF}$$

$Z_{0LCest}(F_{NEW}) = 51.028$ ohms

Compare with:

$Z_{0real} = 51.011$ ohms

Calculation of Z_{0LC} x C x VF "constant":

$$\frac{10^6}{c_{English}} = 1016.703$$

Estimate of complex characteristic impedance from the "real" characteristic impedance and the propagation constant at the new frequency.

The estimate is useful in the HF range.

$$Z_{0est}(f) := Z_{0LC}(f) \cdot \left(1 - j \cdot \frac{\alpha(f)}{\beta(f)}\right)$$

$Z_{0est}(F_{NEW}) = 51.011 - 0.484j$ ohms

Compare with:

$Z_0(F_{NEW}) = 51.013 - 0.464j$ ohms

Estimate of inductance per foot at the new frequency, from an estimate of the physical characteristic impedance and C.

$$L_{est}(f) := \frac{C(f) \cdot Z_{0LCest}(f)^2}{10^6}$$

$L_{est}(F_{NEW}) = 0.08033$ µH/foot

Compare with:

$L(F_{NEW}) = 0.080275$ µH/foot

Estimate of characteristic impedance and eighth-wavelength frequency from impedance magnitude measurements.

The cable is terminated with an AI1H Geometric Resistance Box and impedance magnitude is measured at other end of cable. For this analysis the length of cable is changed so it is an eighth wavelength at the new frequency. The error is expressed as percent departure from the magnitude of the characteristic impedance.

$i := 0 .. 12$ $x_i := 2^{i-1}$ $Z_{L_i} := 1.5625 \cdot x_i$ $Z_{L_0} := 0$ $Z_{L_{12}} := 10^{10}$

$F_{NEW} = 28.8$ MHz $Length_{eighth}(F_{NEW}) = 2.757$ feet

$$Z_{IN_i} := \frac{Z_{L_i} + Z_0(F_{NEW}) \cdot \tanh(Length_{eighth}(F_{NEW}) \cdot \gamma(F_{NEW}))}{1 + \dfrac{Z_{L_i} \cdot \tanh(Length_{eighth}(F_{NEW}) \cdot \gamma(F_{NEW}))}{Z_0(F_{NEW})}}$$

$$Error_i := \frac{\left|Z_{IN_i}\right| - \left|Z_0(F_{NEW})\right|}{\left|Z_0(F_{NEW})\right|} \cdot 100$$

Z_{L_i} = (ohms)	Z_{IN_i} = (ohms)	$\left\|Z_{IN_i}\right\|$ = (ohms)	$Error_i$ = (%)
0	1.2+51i	51	0
1.5625	4.3+50.9i	51	0.1
3.125	7.4+50.5i	51.1	0.1
6.25	13.5+49.3i	51.1	0.2
12.5	24.7+44.9i	51.2	0.4
25	41.2+30.7i	51.4	0.7
50	51.5+0.6i	51.5	0.9
100	41.6-30.2i	51.4	0.7
200	24.7-44.9i	51.2	0.4
400	13.1-49.4i	51.1	0.2
800	6.8-50.6i	51.1	0.1
1600	3.5-50.9i	51	0.1
$1 \cdot 10^{10}$	0.3-51i	51	0

Matched loss of cable of new length at the new frequency

$MtchdLoss(f) := \frac{Length_{NEW}}{100} \cdot A_0(f)$ $MtchdLoss(F_{NEW}) = 1.173462$ dB

Matched loss from reflection coefficient (Z_0 reference) at the new frequency when termination is an open circuit. This is the method of halving the return loss. This is an exact result when the complex value of the characteristic impedance is known.

$Z_{OC}(f) := \frac{Z_0(f)}{\tanh(Length_{NEW} \cdot \gamma(f))}$ $\rho_{OC}(f) := \frac{Z_{OC}(f) - Z_0(f)}{Z_{OC}(f) + Z_0(f)}$

$MtchdLoss_{OC}(f) := -10 \cdot \log(|\rho_{OC}(f)|)$ $MtchdLoss_{OC}(F_{NEW}) = 1.173461581$ dB

Compare with: $MtchdLoss(F_{NEW}) = 1.173461581$ dB

Matched loss from reflection coefficient (Z_0 reference) at the new frequency when termination is a short circuit. This is the method of halving the return loss. This is an exact result when the complex value of the characteristic impedance is known.

$Z_{SC}(f) := Z_0(f) \cdot \tanh(Length_{NEW} \cdot \gamma(f))$ $\rho_{SC}(f) := \frac{Z_{SC}(f) - Z_0(f)}{Z_{SC}(f) + Z_0(f)}$

$MtchdLoss_{SC}(f) := -10 \cdot \log(|\rho_{SC}(f)|)$ $MtchdLoss_{SC}(F_{NEW}) = 1.173461581$ dB

Compare with: $MtchdLoss(F_{NEW}) = 1.173461581$ dB

Matched loss from reflection coefficient ("real" characteristic impedance reference) at the new frequency when termination is an open circuit. This is the method of halving the return loss.

$\rho_{OCapprox}(f) := \frac{Z_{OC}(f) - Z_{0real}}{Z_{OC}(f) + Z_{0real}}$ $MtchdLoss_{OCapprox}(f) := -10 \cdot \log(|\rho_{OCapprox}(f)|)$

$MtchdLoss_{OCapprox}(F_{NEW}) = 1.182134$ dB Compare with: $MtchdLoss(F_{NEW}) = 1.173462$ dB

Matched loss from reflection coefficient ("real" characteristic impedance reference) at the new frequency when termination is a short circuit. This is the method of halving the return loss.

$\rho_{SCapprox}(f) := \frac{Z_{SC}(f) - Z_{0real}}{Z_{SC}(f) + Z_{0real}}$ $MtchdLoss_{SCapprox}(f) := -10 \cdot \log(|\rho_{SCapprox}(f)|)$

$MtchdLoss_{SCapprox}(F_{NEW}) = 1.164698$ dB Compare with: $MtchdLoss(F_{NEW}) = 1.173462$ dB

Matched loss at the new frequency by geometric averaging reflection coefficients ("real" characteristic impedance reference)

$MtchdLoss_{approx}(f) := -5 \cdot \log(|\rho_{SCapprox}(f)| \cdot |\rho_{OCapprox}(f)|)$

$MtchdLoss_{approx}(F_{NEW}) = 1.173416$ dB Compare with: $MtchdLoss(F_{NEW}) = 1.173462$ dB

Minimum loss at the new frequency by terminating the cable in the complex conjugate of Z_0 calculated using the abcd matrix method. This loss is lower than the matched loss.

$a(f) := \cosh\left(\gamma(f)\cdot Length_{NEW}\right)$ $\quad$ $b(f) := Z_0(f)\cdot\sinh\left(\gamma(f)\cdot Length_{NEW}\right)$

$c(f) := \dfrac{\sinh\left(\gamma(f)\cdot Length_{NEW}\right)}{Z_0(f)}$ $\quad$ $d(f) := \cosh\left(\gamma(f)\cdot Length_{NEW}\right)$

Assume that 1 ampere peak flows in the load: $I_L = 1$

$Z_L(f) := \overline{Z_0(f)}$ $\quad$ $\overline{Z_0\left(F_{NEW}\right)} = 51.013 + 0.464j$ ohms

$E_L(f) := Z_L(f)$ $\quad$ $P_L(f) := \dfrac{Re\left(Z_L(f)\right)}{2}$

$E_{IN}(f) := a(f)\cdot E_L(f) + b(f)$ $\quad$ $I_{IN}(f) := c(f)\cdot E_L(f) + d(f)$

$Z_{IN}(f) := \dfrac{E_{IN}(f)}{I_{IN}(f)}$ $\quad$ $P_{IN}(f) := \dfrac{\left(\left|I_{IN}(f)\right|\right)^2}{2}\cdot Re\left(Z_{IN}(f)\right)$

$MinLoss_{abcd}(f) := 10\cdot\log\left(\dfrac{P_{IN}(f)}{P_L(f)}\right)$ $\quad$ $MinLoss_{abcd}\left(F_{NEW}\right) = 1.1723563694$ dB

Compare with: $MtchdLoss\left(F_{NEW}\right) = 1.173461581$ dB, which is **greater** than the minimum loss

Now we calculate the standing wave ratio on the line:

$\rho_{Load}(f) := \dfrac{\overline{Z_0(f)} - Z_0(f)}{\overline{Z_0(f)} + Z_0(f)}$ $\quad$ $SWR(f) := \dfrac{1 + \left|\rho_{Load}(f)\right|}{1 - \left|\rho_{Load}(f)\right|}$ $\quad$ $SWR\left(F_{NEW}\right) = 1.018$

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TRANSMISSION LINE CALCULATIONS FOR ANY FREQUENCY AND LENGTH

WHEN THE LOAD IMPEDANCE IS KNOWN

Loss, SWR and input impedance for any frequency, length and load impedance. The subscript "1" is used for this calculation. Modify the input data as desired.

Input data:

$F_1 := 14$ MHz $\quad$ $Length_1 := 100$ feet $\quad$ $Z_{L1} := 50 - 500 \cdot j$ ohms

Calculations:

$$Z_{IN1}(f) := \frac{Z_{L1} + Z_0(f) \cdot \tanh\left(Length_1 \cdot \gamma(f)\right)}{1 + \dfrac{Z_{L1} \cdot \tanh\left(Length_1 \cdot \gamma(f)\right)}{Z_0(f)}}$$

Modified reflection coefficients at load and input:

$$\rho_{LM1}(f) := \frac{Z_{L1} - \overline{Z_0(f)}}{Z_{L1} + Z_0(f)} \qquad \rho_{INM1}(f) := \frac{Z_{IN1}(f) - \overline{Z_0(f)}}{Z_{IN1}(f) + Z_0(f)}$$

Matched loss at F_1:

$$MtchdLoss_1 := \frac{Length_1}{100} \cdot A_0\left(F_1\right)$$

Total loss at F_1:

$$TotalLoss_1 := MtchdLoss_1 + 10 \cdot \log\left[\frac{1 - \left(\left|\rho_{INM1}\left(F_1\right)\right|\right)^2}{1 - \left(\left|\rho_{LM1}\left(F_1\right)\right|\right)^2}\right]$$

Classical reflection coefficients and SWR at load and input:

$$\rho_{L1}(f) := \frac{Z_{L1} - Z_0(f)}{Z_{L1} + Z_0(f)} \qquad \rho_{IN1}(f) := \frac{Z_{IN1}(f) - Z_0(f)}{Z_{IN1}(f) + Z_0(f)}$$

$$SWR_{L1}(f) := \frac{1 + \left|\rho_{L1}(f)\right|}{1 - \left|\rho_{L1}(f)\right|} \qquad SWR_{IN1}(f) := \frac{1 + \left|\rho_{IN1}(f)\right|}{1 - \left|\rho_{IN1}(f)\right|}$$

Summary:

$F_1 = 14$ MHz

$Length_1 = 100$ feet

$Z_{L1} = 50 - 500j$ ohms

$MtchdLoss_1 = 1.63$ dB

$TotalLoss_1 = 13.04$ dB

$SWR_{L1}\left(F_1\right) = 88.39$

$SWR_{IN1}\left(F_1\right) = 5.1$

$Z_{IN1}\left(F_1\right) = 10.2251 - 9.5109j$ ohms

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WHEN THE INPUT IMPEDANCE IS KNOWN

Loss, SWR and load impedance for any frequency, length and input impedance. The subscript "2" is used for this calculation. Modify the input data as desired.

Input data:

$F_2 := 14$ MHz $Length_2 := 100$ feet $Z_{IN2} := 10.2251 - 9.5109 \cdot j$ ohms

Calculations:

$$Z_{L2}(f) := \frac{Z_{IN2} - Z_0(f) \cdot \tanh\left(Length_2 \cdot \gamma(f)\right)}{1 - \dfrac{Z_{IN2} \cdot \tanh\left(Length_2 \cdot \gamma(f)\right)}{Z_0(f)}}$$

Modified reflection coefficients at load and input:

$$\rho_{LM2}(f) := \frac{Z_{L2}(f) - \overline{Z_0(f)}}{Z_{L2}(f) + Z_0(f)} \qquad \rho_{INM2}(f) := \frac{Z_{IN2} - \overline{Z_0(f)}}{Z_{IN2} + Z_0(f)}$$

Matched loss at F_2:

$$MtchdLoss_2 := \frac{Length_2}{100} \cdot A_0\left(F_2\right)$$

Total loss at F_2:

$$TotalLoss_2 := MtchdLoss_2 + 10 \cdot \log\left[\frac{1 - \left(\left|\rho_{INM2}\left(F_2\right)\right|\right)^2}{1 - \left(\left|\rho_{LM2}\left(F_2\right)\right|\right)^2}\right]$$

Classical reflection coefficients and SWR at load and input:

$$\rho_{L2}(f) := \frac{Z_{L2}(f) - Z_0(f)}{Z_{L2}(f) + Z_0(f)} \qquad \rho_{IN2}(f) := \frac{Z_{IN2} - Z_0(f)}{Z_{IN2} + Z_0(f)}$$

$$SWR_{L2}(f) := \frac{1 + \left|\rho_{L2}(f)\right|}{1 - \left|\rho_{L2}(f)\right|} \qquad SWR_{IN2}(f) := \frac{1 + \left|\rho_{IN2}(f)\right|}{1 - \left|\rho_{IN2}(f)\right|}$$

Summary:

$F_2 = 14$ MHz

$Length_2 = 100$ feet

$Z_{IN2} = 10.2251 - 9.5109j$ ohms

$MtchdLoss_2 = 1.63$ dB

$TotalLoss_2 = 13.04$ dB

$SWR_{L2}\left(F_2\right) = 88.38$

$SWR_{IN2}\left(F_2\right) = 5.1$

$Z_{L2}\left(F_2\right) = 50 - 500j$ ohms

Alternative loss calculation for the same example using the abcd matrix representation of the transmission line.

$a(f) := \cosh\left(\gamma(f)\cdot Length_2\right)$ $\qquad$ $b(f) := Z_0(f)\cdot\sinh\left(\gamma(f)\cdot Length_2\right)$

$c(f) := \dfrac{\sinh\left(\gamma(f)\cdot Length_2\right)}{Z_0(f)}$ $\qquad$ $d(f) := \cosh\left(\gamma(f)\cdot Length_2\right)$

Assume that 1 ampere peak flows in the load: $I_L = 1$ $\qquad$ Use Z_{L2} from previous page.

$E_L(f) := Z_{L2}(f)$ $\qquad$ $E_{IN}(f) := a(f)\cdot E_L(f) + b(f)$ $\qquad$ $I_{IN}(f) := c(f)\cdot E_L(f) + d(f)$

Check:

$Z_{IN2} := \dfrac{E_{IN}\left(F_2\right)}{I_{IN}\left(F_2\right)}$ $\qquad$ $Z_{IN2} = 10.2251 - 9.5109j$ ohms

$P_{L2}(f) := \dfrac{\mathrm{Re}\left(Z_{L2}(f)\right)}{2}$ $\qquad$ $P_{IN2}(f) := \dfrac{\left(\left|I_{IN}(f)\right|\right)^2}{2}\cdot\mathrm{Re}\left(Z_{IN2}\right)$

$TotalLoss_{abcd2} := 10\cdot\log\left(\dfrac{P_{IN2}\left(F_2\right)}{P_{L2}\left(F_2\right)}\right)$ $\qquad$ $TotalLoss_{abcd2} = 13.0377260849$ dB

Compare with: $TotalLoss_2 = 13.0377260849$ dB

Approximate loss calculation using real characteristic impedance. For very high loss situations, the error incurred by not using the complex characteristic impedance and the modified reflection coefficient can be significant.

$\rho_{Lreal2}(f) := \dfrac{Z_{L2}(f) - Z_{0real}}{Z_{L2}(f) + Z_{0real}}$ $\qquad$ $\rho_{INreal2}(f) := \dfrac{Z_{IN2} - Z_{0real}}{Z_{IN2} + Z_{0real}}$

$$TotalLoss_{real2} := MtchdLoss_2 + 10\cdot\log\left[\frac{1 - \left(\left|\rho_{INreal2}\left(F_2\right)\right|\right)^2}{1 - \left(\left|\rho_{Lreal2}\left(F_2\right)\right|\right)^2}\right]$$

$TotalLoss_{real2} = 13.0420721373$ dB

Compare with: $TotalLoss_2 = 13.0377260849$ dB

SERIES RESONATOR DESIGN

The length is changed to be a quarter wavelength or a half wavelength at the design frequency. The subscript QOC means quarter-wave open-circuited. The subscript HSC means half-wave short-circuited.

$F_{SR} := 21\ \mathrm{MHz}$

Open-circuited quarter-wavelength segment

$\mathrm{Length}_{quarter}\left(F_{SR}\right) = 7.562$ feet

$$Z_{INQOC}(f) := \frac{Z_0(f)}{\tanh\left(\mathrm{Length}_{quarter}\left(F_{SR}\right)\cdot\gamma(f)\right)}$$

$Z_{INQOC}\left(F_{SR}\right) = 0.887 - 9.483j\cdot 10^{-3}$ ohms

$\left|Z_{INQOC}\left(F_{SR}\right)\right| = 0.887$ ohms

$$X_{QOC}(f) := \frac{f}{2}\cdot\frac{d}{df}\mathrm{Im}\left(Z_{INQOC}(f)\right)$$

$X_{QOC}\left(F_{SR}\right) = 40.05$ ohms

$$Q_{QOC}(f) := \frac{X_{QOC}(f)}{\mathrm{Re}\left(Z_{INQOC}(f)\right)}$$

$Q_{QOC}\left(F_{SR}\right) = 45.14$

Short-circuited half-wavelength segment

$\mathrm{Length}_{half}\left(F_{SR}\right) = 15.125$ feet

$$Z_{INHSC}(f) := Z_0(f)\cdot\tanh\left(\mathrm{Length}_{half}\left(F_{SR}\right)\cdot\gamma(f)\right)$$

$Z_{INHSC}\left(F_{SR}\right) = 1.774 - 0.019j$ ohms

$\left|Z_{INHSC}\left(F_{SR}\right)\right| = 1.774$ ohms

$$X_{HSC}(f) := \frac{f}{2}\cdot\frac{d}{df}\mathrm{Im}\left(Z_{INHSC}(f)\right)$$

$X_{HSC}\left(F_{SR}\right) = 80.04$ ohms

$$Q_{HSC}(f) := \frac{X_{HSC}(f)}{\mathrm{Re}\left(Z_{INHSC}(f)\right)}$$

$Q_{HSC}\left(F_{SR}\right) = 45.11$

Compare with the following approximations:

$$X_{QOCest}(f) := \frac{Z_{0real}\cdot\pi}{4}$$

$X_{QOCest}\left(F_{SR}\right) = 40.06$ ohms

$$X_{HSCest}(f) := \frac{Z_{0real}\cdot\pi}{2}$$

$X_{HSCest}\left(F_{SR}\right) = 80.13$ ohms

$k_3 = 2.774332$

$$Q_{QOCest}(f) := \frac{k_3\cdot f}{A_0(f)\cdot VF}$$

$Q_{QOCest}\left(F_{SR}\right) = 45.15$

$$Q_{HSCest}(f) := \frac{k_3\cdot f}{A_0(f)\cdot VF}$$

$Q_{HSCest}\left(F_{SR}\right) = 45.15$

PARALLEL RESONATOR DESIGN

The length is changed to be a quarter wavelength or a half wavelength at the design frequency. The subscript HOC means half-wave open-circuited. The subscript QSC means quarter-wave short-circuited.

$F_{PR} := 21\ \text{MHz}$

Open-circuited half-wavelength segment

$Length_{half}(F_{PR}) = 15.125$ feet

$$Z_{INHOC}(f) := \frac{Z_0(f)}{\tanh\left(Length_{half}(F_{PR})\cdot\gamma(f)\right)}$$

$Z_{INHOC}(F_{PR}) = 1466.89 - 15.68j$ ohms

$\left|Z_{INHOC}(F_{PR})\right| = 1466.97$ ohms

$$B_{HOC}(f) := \frac{f}{2}\cdot\frac{d}{df}\mathrm{Im}\left(\frac{1}{Z_{INHOC}(f)}\right)$$

$B_{HOC}(F_{PR}) = 0.030751$ siemens

$$Q_{HOC}(f) := \frac{B_{HOC}(f)}{\mathrm{Re}\left(\frac{1}{Z_{INHOC}(f)}\right)}$$

$Q_{HOC}(F_{PR}) = 45.11$

Short-circuited quarter-wavelength segment

$Length_{quarter}(F_{PR}) = 7.562$ feet

$$Z_{INQSC}(f) := Z_0(f)\cdot\tanh\left(Length_{quarter}(F_{PR})\cdot\gamma(f)\right)$$

$Z_{INQSC}(F_{PR}) = 2932.89 - 31.34j$ ohms

$\left|Z_{INQSC}(F_{PR})\right| = 2933.05$ ohms

$$B_{QSC}(f) := \frac{f}{2}\cdot\frac{d}{df}\mathrm{Im}\left(\frac{1}{Z_{INQSC}(f)}\right)$$

$B_{QSC}(F_{PR}) = 0.015389$ siemens

$$Q_{QSC}(f) := \frac{B_{QSC}(f)}{\mathrm{Re}\left(\frac{1}{Z_{INQSC}(f)}\right)}$$

$Q_{QSC}(F_{PR}) = 45.14$

Compare with the following approximations:

$$B_{HOCest}(f) := \frac{\pi}{2\cdot Z_{0real}}$$

$B_{HOCest}(F_{PR}) = 0.030793$ seimens

$$B_{QSCest}(f) := \frac{\pi}{4\cdot Z_{0real}}$$

$B_{QSCest}(F_{PR}) = 0.015397$ seimens

$k_3 = 2.774332$

$$Q_{HOCest}(f) := \frac{k_3\cdot f}{A_0(f)\cdot VF}$$

$Q_{HOCest}(F_{PR}) = 45.15$

$$Q_{QSCest}(f) := \frac{k_3\cdot f}{A_0(f)\cdot VF}$$

$Q_{QSCest}(F_{PR}) = 45.15$

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INDUCTIVE AND CAPACITIVE REACTANCE DESIGN

The lengths required to achieve the desired inductive or capacitive reactance at any frequency are calculated. The lengths are the shortest ones that will achieve the desired reactance. The subscript L applies to the inductive reactance and the subscript C applies to the capacitive reactance. The inductive reactance occurs when the far end is shorted and the capacitive reactance is achieved when the far end is open circuited. Substitute the desired frequency and reactance for the ones shown.

$F_{LC} := 21\ \mathrm{MHz}$ $\qquad$ $X_{desired} := 100$ ohms

Inductive reactance
Shorted segment

$$Length_L := \frac{\operatorname{atan}\left[\dfrac{X_{desired}}{\left(R_0\left(F_{LC}\right)\right)}\right]}{\beta\left(F_{LC}\right)}$$

$Length_L = 5.29$ feet

$$Z_L := Z_0\left(F_{LC}\right)\cdot\tanh\left(Length_L\cdot\gamma\left(F_{LC}\right)\right)$$

$Z_L = 4.1 + 99.9j$ ohms

$$L_{effective} := \frac{\operatorname{Im}\left(Z_L\right)}{2\cdot\pi\cdot F_{LC}}$$

$L_{effective} = 0.757$ µH

$$Q_L := \frac{\operatorname{Im}\left(Z_L\right)}{\operatorname{Re}\left(Z_L\right)}$$

$Q_L = 24.5$

Capacitive reactance
Open-circuited segment

$$Length_C := \frac{\operatorname{atan}\left(\dfrac{R_0\left(F_{LC}\right)}{X_{desired}}\right)}{\beta\left(F_{LC}\right)}$$

$Length_C = 2.27$ feet

$$Z_C := \frac{Z_0\left(F_{LC}\right)}{\tanh\left(Length_C\cdot\gamma\left(F_{LC}\right)\right)}$$

$Z_C = 0.2 - 100j$ ohms

$$C_{effective} := \frac{\operatorname{Im}\left(\dfrac{1}{Z_C}\right)}{2\cdot\pi\cdot F_{LC}\cdot 10^{-6}}$$

$C_{effective} = 75.79$ pF

$$Q_C := \frac{-\operatorname{Im}\left(Z_C\right)}{\operatorname{Re}\left(Z_C\right)}$$

$Q_C = 450.7$