TRANSMISSION LINE PROPERTIES FROM MANUFACTURER'S DATA

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See "Transmission Line Properties from Manufacturer's Data" by Frank Witt, AI1H
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PHYSICAL CONSTANTS AND CONVERSION FACTORS

$c_{Metric} := 299.792458$ Megameters/sec $\qquad k_1 := .3048$ meters/foot

$c_{English} := \frac{c_{Metric}}{k_1}$ $\qquad c_{English} = 983.571056$ Megafeet/sec

$k_2 := \frac{\ln(10)}{20}$ $\qquad k_2 = 0.1151292546$ nepers/dB

MANUFACTURER'S DATA

Substitute data for other transmission lines. Additional data are available in TLMANDATAV8.MCD. Delete the data below on this page and copy and paste desired data from TLMANDATAV8.MCD in its place.

Description:		
	Manufacturer:	Belden
	Type:	RG-58A/U
	Trade No.	8259
	Insulation:	Solid polyethylene

Electrical data: A_0 = Matched Loss $\qquad Z_{0real}$ = "real" characteristic impedance

ND is the number of matched loss data points: $ND := 9 \qquad i := 1..ND$

$F_i :=$ MHz	$A_{0_i} :=$ dB/100 feet
1	.44
10	1.4
50	3.3
100	4.9
200	7.3
400	11.5
700	17.0
900	20.0
1000	21.5

$Z_{0real} := 50$ ohms

$VF_{nominal} := .66$

$C_{nominal} := 30.8$ pF/foot

$V_{max} := 1400$ volts rms

$T_{maxC} := 75$ degrees C

$Diameter := .193$ inches

$F_{LOW} := F_3 \qquad F_{HIGH} := F_{ND-1}$

$A_{0LOW} := A_{0_3} \qquad A_{0HIGH} := A_{0_{ND-1}}$

Assumed frequency dependence of loss due to insulation: $g := 1.1$

TRANSMISSION LINE PROPERTIES

CONSISTENCY CHECK
Calculation of Z_{0real} x C x VF "constant":

$$\frac{10^6}{c_{English}} = 1016.703 \qquad Z_{0real} \cdot C_{nominal} \cdot VF_{nominal} = 1016.4$$

Correct velocity factor:

$$VF := \frac{10^6}{c_{English} \cdot Z_{0real} \cdot C_{nominal}} \qquad VF = 0.660197$$

Compare with: $VF_{nominal} = 0.66$

$$Consistency := \mathrm{if}\left(VF > VF_{nominal}, \frac{VF_{nominal} \cdot 100}{VF}, \frac{VF \cdot 100}{VF_{nominal}}\right) \qquad Consistency = 99.97 \quad \%$$

Use the corrected velocity factor for consistency. Use the nominal values for the capacitance per foot and "real" characteristic impedance. Take Z_{0real} to be the physical characteristic impedance, Z_{0LC}, which depends only on the inductance and capacitance per foot, which are assumed to be independent of frequency over the frequency range of interest.

$VF = 0.660197$

$C := C_{nominal}$ $\qquad C = 30.8$ pF/foot

$Z_{0LC} := Z_{0real}$ $\qquad Z_{0LC} = 50$ ohms

FREQUENCY DEPENDENCE OF MATCHED LOSS

Assume that the matched loss due to the conductors increases as the square root of frequency and the matched loss due to the insulation increases in proportion to the frequency raised to the power g. From the total matched losses at a low and a high frequency the conductor and insulation components are determined. Then the frequency dependent matched loss components are found.

$$A_{0CONDLOW} := \frac{F_{LOW}^{\,g} \cdot A_{0HIGH} - F_{HIGH}^{\,g} \cdot A_{0LOW}}{F_{LOW}^{\,g} \cdot \sqrt{\frac{F_{HIGH}}{F_{LOW}}} - F_{HIGH}^{\,g}}$$

$$A_{0INSLOW} := A_{0LOW} - A_{0CONDLOW}$$

$$A_{0COND}(f) := A_{0CONDLOW} \cdot \sqrt{\frac{f}{F_{LOW}}}$$

$$A_{0INS}(f) := A_{0INSLOW} \left(\frac{f}{F_{LOW}}\right)^g$$

$$A_{0TOT}(f) := A_{0COND}(f) + A_{0INS}(f)$$

$$Error_i := A_{0TOT}(F_i) - A_{0_i}$$

Comparison of calculated matched loss with manufacturer's data

$F_i =$ (MHz)	$A_{0_i} =$ (dB)	$A_{0TOT}(F_i)$ (dB)	$Error_i =$ (dB)
1	0.4	0.4	-0
10	1.4	1.4	$-8.2\cdot10^{-3}$
50	3.3	3.3	0
100	4.9	4.9	-0
200	7.3	7.4	0.1
400	11.5	11.5	-0
700	17	16.7	-0.3
900	20	20	$-7.1\cdot10^{-15}$
$1\cdot10^3$	21.5	21.6	0.1

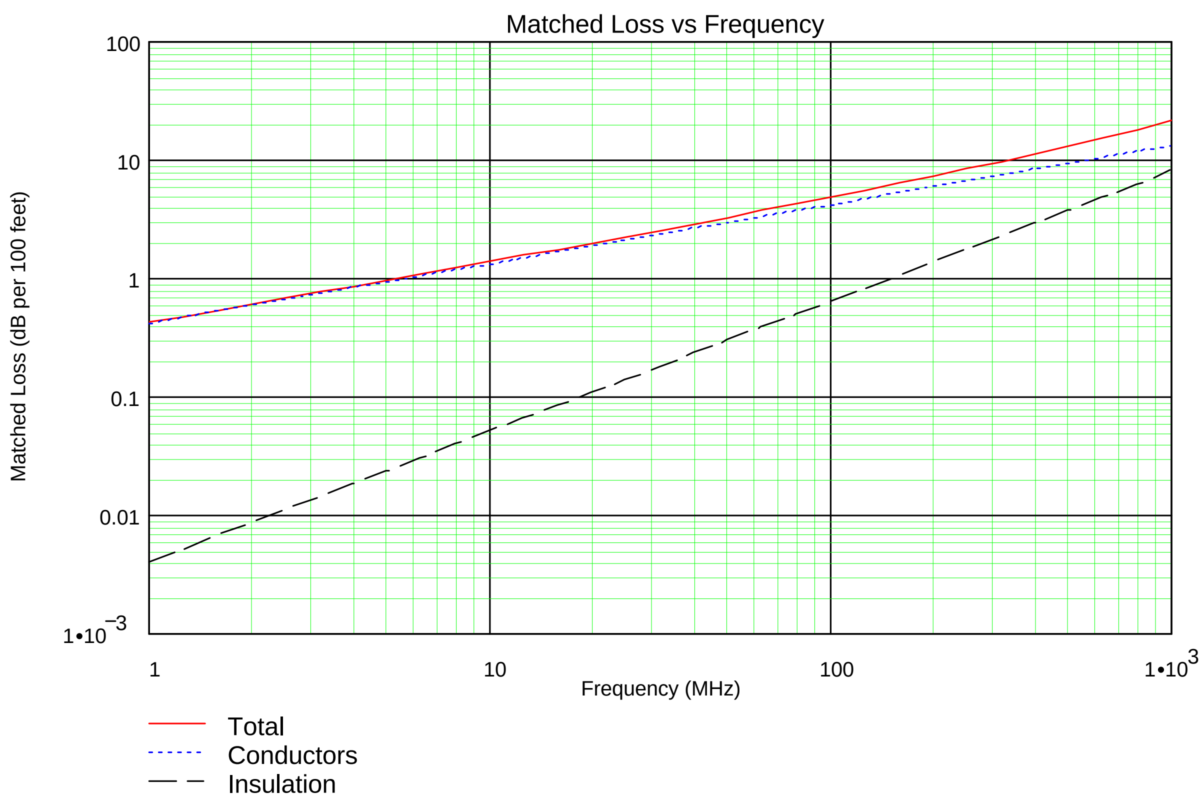
$$Error_{RMS} := \frac{1}{ND}\cdot\sqrt{\sum_i \left(Error_i\right)^2}$$

$Error_{RMS} = 0.032$ dB

$g = 1.1$

$j := 0..30$

$f_j := 10^{\frac{j}{10}}$



CROSSOVER FREQUENCY

This is the frequency where the loss due to the conductors is equal to the loss due the insulation.

$$F_X := \text{if}\left[g \le .5, 0, F_{LOW}\cdot\left(\frac{A_{0INSLOW}}{A_{0CONDLOW}}\right)^{\frac{2}{1-2\cdot g}}\right] \qquad F_X = 2277 \quad \text{MHz}$$

PROPAGATION CONSTANT AS A FUNCTION OF FREQUENCY

$$\alpha(f) := \frac{k_2}{100}\cdot A_{0TOT}(f) \qquad \beta(f) := \frac{2\cdot\pi\cdot f}{VF\cdot c_{English}} \qquad \gamma(f) := \alpha(f) + \beta(f)\cdot j$$

COMPLEX CHARACTERISTIC IMPEDANCE AS A FUNCTION OF FREQUENCY

Calculation of R(f), G(f) and Z_0(f) are estimates because the frequency dependence of the characteristic impedance is not known at this point.

$$\alpha_{COND}(f) := \frac{k_2}{100}\cdot A_{0COND}(f) \qquad R_{est}(f) := 2\cdot Z_{0LC}\cdot\alpha_{COND}(f)$$

$$\alpha_{INS}(f) := \frac{k_2}{100}\cdot A_{0INS}(f) \qquad G_{est}(f) := \frac{2\cdot\alpha_{INS}(f)}{Z_{0LC}}$$

$$C(f) := C \qquad L(f) := \frac{C(f)\cdot Z_{0LC}^{\,2}}{10^6} \qquad Z_{0est}(f) := \sqrt{\frac{R_{est}(f) + 2\cdot\pi\cdot f\cdot L(f)\cdot j}{G_{est}(f) + 2\cdot\pi\cdot f\cdot 10^{-6}\cdot C(f)\cdot j}}$$

Calculation of R(f), G(f) and Z_0(f). This is an iteration that recognizes the impact of the frequency dependence of Z_0 on R and G.

$$R(f) := 2\cdot\mathrm{Re}\left(Z_{0est}(f)\right)\cdot\alpha_{COND}(f) \qquad G(f) := \frac{2\cdot\alpha_{INS}(f)\cdot\mathrm{Re}\left(Z_{0est}(f)\right)}{\left(\left|Z_{0est}(f)\right|\right)^2}$$

$$Z_0(f) := \sqrt{\frac{R(f) + 2\cdot\pi\cdot f\cdot L(f)\cdot j}{G(f) + 2\cdot\pi\cdot f\cdot 10^{-6}\cdot C(f)\cdot j}}$$

This final iteration of Z_0(f) uses the new version of Z_0(f) to find R(f) and G(f).

$$R(f) := 2\cdot\mathrm{Re}\left(Z_0(f)\right)\cdot\alpha_{COND}(f) \qquad G(f) := \frac{2\cdot\alpha_{INS}(f)\cdot\mathrm{Re}\left(Z_0(f)\right)}{\left(\left|Z_0(f)\right|\right)^2}$$

$$Z_0(f) := \sqrt{\frac{R(f) + 2\cdot\pi\cdot f\cdot L(f)\cdot j}{G(f) + 2\cdot\pi\cdot f\cdot 10^{-6}\cdot C(f)\cdot j}}$$

$$R_0(f) := \mathrm{Re}\left(Z_0(f)\right)$$
$$X_0(f) := \mathrm{Im}\left(Z_0(f)\right)$$

SOLUTIONS FOR AN ARBITRARY FREQUENCY AND LENGTH

Frequency and length. Replace these with the desired frequency and length.

$F := 28.8\ \text{MHz}$ $\qquad$ $Length := 50\ \text{feet}$

Electrical angle and length in wavelengths at chosen frequency and length

$\theta(f) := \beta(f) \cdot Length \cdot \dfrac{180}{\pi}$ $\qquad$ $\theta(F) = 798.34$ degrees

$Wavelengths(f) := \dfrac{Length \cdot f}{c_{English} \cdot VF}$ $\qquad$ $Wavelengths(F) = 2.218$

Interesting lengths - Cable lengths required to create half, quarter and eighth wavelength at chosen frequency

$Length_{half}(f) := \dfrac{c_{English} \cdot VF}{2 \cdot f}$ $\qquad$ $Length_{half}(F) = 11.27$ feet

$Length_{quarter}(f) := \dfrac{Length_{half}(f)}{2}$ $\qquad$ $Length_{quarter}(F) = 5.64$ feet

$Length_{eighth}(f) := \dfrac{Length_{half}(f)}{4}$ $\qquad$ $Length_{eighth}(F) = 2.82$ feet

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Interesting frequencies - Frequencies at which the cable segment is a half, quarter and eighth wavelength.

$F_{half} := \dfrac{c_{English} \cdot VF}{2 \cdot Length}$ $\qquad$ $F_{half} = 6.494$ MHz

$F_{quarter} := \dfrac{F_{half}}{2}$ $\qquad$ $F_{quarter} = 3.247$ MHz

$F_{eighth} := \dfrac{F_{half}}{4}$ $\qquad$ $F_{eighth} = 1.623$ MHz

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Characteristic impedance at the chosen frequency

$Z_0(F) = 50.002 - 0.436i$ ohms

Propagation constant at the chosen frequency. $\gamma(F) := \alpha(F) + \beta(F) \cdot j$

$\alpha(F) = 2.809 \cdot 10^{-3}$ nepers/foot

$\beta(F) = 0.279$ radians/foot

Resistance, inductance, conductance and capacitance per foot at the chosen frequency

$R(f) := \mathrm{Re}\left(\gamma(f)\cdot Z_0(f)\right)$ $\qquad R(F) = 0.2619$ ohms/foot

$L(f) := \dfrac{\mathrm{Im}\left(\gamma(f)\cdot Z_0(f)\right)}{2\cdot\pi\cdot f}$ $\qquad L(F) = 0.077$ µH/foot

$G(f) := \mathrm{Re}\left(\dfrac{\gamma(f)}{Z_0(f)}\right)$ $\qquad G(F) = 7.611\cdot 10^{-6}$ siemens/foot

$C(f) := \dfrac{\mathrm{Im}\left(\dfrac{\gamma(f)}{Z_0(f)}\right)}{2\cdot\pi\cdot f\cdot 10^{-6}}$ $\qquad C(F) = 30.8$ pF/foot

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Matched loss of cable of chosen length at the chosen frequency

$\mathrm{MtchdLoss}(f) := \dfrac{\mathrm{Length}}{100}\cdot A_{0TOT}(f)$

$\mathrm{MtchdLoss}(F) = 1.22$ dB

TRANSMISSION LINE CALCULATIONS FOR ANY FREQUENCY, LENGTH AND INPUT POWER

WHEN THE LOAD IMPEDANCE IS KNOWN

Loss, SWR, input impedance and voltage and power stress for any frequency, length, load impedance and power input. The subscript "1" is used for this calculation. Modify the input data as desired.

Input data:

$F_1 := 14\ \mathrm{MHz}$

$\mathrm{Length}_1 := 100\ \mathrm{feet}$

$Z_{L1} := 50 - 500 \cdot j\ \ \mathrm{ohms}$

$P_{IN1} := 1500\ \mathrm{watts}$

Calculations:

$$Z_{IN1}(f) := \frac{Z_{L1} + Z_0(f) \cdot \tanh\left(\mathrm{Length}_1 \cdot \gamma(f)\right)}{1 + \dfrac{Z_{L1} \cdot \tanh\left(\mathrm{Length}_1 \cdot \gamma(f)\right)}{Z_0(f)}}$$

Modified reflection coefficients at load and input:

$$\rho_{LM1}(f) := \frac{Z_{L1} - \overline{Z_0(f)}}{Z_{L1} + Z_0(f)}$$

$$\rho_{INM1}(f) := \frac{Z_{IN1}(f) - \overline{Z_0(f)}}{Z_{IN1}(f) + Z_0(f)}$$

Matched loss at F_1:

$$\mathrm{MtchdLoss}_1 := \frac{\mathrm{Length}_1}{100} \cdot A_{0TOT}\left(F_1\right)$$

Total loss at F_1:

$$\mathrm{TotalLoss}_1 := \mathrm{MtchdLoss}_1 + 10 \cdot \log\left[\frac{1 - \left(\left|\rho_{INM1}\left(F_1\right)\right|\right)^2}{1 - \left(\left|\rho_{LM1}\left(F_1\right)\right|\right)^2}\right]$$

Classical reflection coefficients and SWR at load and input:

$$\rho_{L1}(f) := \frac{Z_{L1} - Z_0(f)}{Z_{L1} + Z_0(f)} \qquad \rho_{IN1}(f) := \frac{Z_{IN1}(f) - Z_0(f)}{Z_{IN1}(f) + Z_0(f)}$$

$$\mathrm{SWR}_{L1}(f) := \frac{1 + \left|\rho_{L1}(f)\right|}{1 - \left|\rho_{L1}(f)\right|} \qquad \mathrm{SWR}_{IN1}(f) := \frac{1 + \left|\rho_{IN1}(f)\right|}{1 - \left|\rho_{IN1}(f)\right|}$$

WHEN THE LOAD IMPEDANCE IS KNOWN (Continued)

Voltage and power stress calculation. We calculate the stress when the input power is P_{IN1}. Break the cable into 100 equal length segments.

$m := 0 .. 100 \qquad x_m := m \cdot \frac{Length_1}{100}$

Assume initially that 1 ampere rms flows in load impedance

$V_1(x) := Z_{L1} \cdot \cosh(\gamma(F_1) \cdot x) + Z_0(F_1) \cdot \sinh(\gamma(F_1) \cdot x)$

$I_1(x) := \cosh(\gamma(F_1) \cdot x) + \frac{Z_{L1}}{Z_0(F_1)} \cdot \sinh(\gamma(F_1) \cdot x)$

$V_{1mag}(x) := |V_1(x)|$

$V_{1mag_m} := V_{1mag}(x_m)$

$P_{1COND}(x) := (|I_1(x)|)^2 \cdot R(F_1)$

$P_{1INS}(x) := (|V_1(x)|)^2 \cdot G(F_1)$

$P_{1TOT}(x) := P_{1COND}(x) + P_{1INS}(x)$

$P_{1TOT_m} := P_{1TOT}(x_m)$

Compute maximum voltage and watts per foot along line when input power equals P_{IN1}

$P_{L1} := P_{IN1} \cdot 10^{\frac{-TotalLoss_1}{10}}$

$R_{L1} := Re(Z_{L1})$

$I_{L1} := \sqrt{\frac{P_{L1}}{R_{L1}}}$

$V_{maxPIN1} := \max(V_{1mag}) \cdot I_{L1}$

$P_{TOT1perft} := I_{L1}^2 \cdot \max(P_{1TOT})$

Check: The sum of the power in the cable segments plus the power in the load should equal the input power. This calculation will not be exact because of the finite number of cable segments.

$$P_{CABLE1} := I_{L1}^2 \cdot \left[\frac{1}{2} \cdot \left(P_{1TOT_0} + P_{1TOT_{100}}\right) + \sum_{m=1}^{99} P_{1TOT_m} \right] \cdot \frac{Length_1}{100}$$

$P_{CABLE1} = 1427.512$ watts

$P_{Input1} := P_{CABLE1} + P_{L1} \qquad P_{Input1} = 1500.187$ watts

Compare with: $\qquad P_{IN1} = 1500$ watts

TRANSMISSION LINE CALCULATIONS FOR ANY FREQUENCY, LENGTH AND INPUT POWER

WHEN THE LOAD IMPEDANCE IS KNOWN

SUMMARY

Input data:

$F_1 = 14$ MHz

$Length_1 = 100$ feet

$Z_{L1} = 50 - 500i$ ohms

$P_{IN1} = 1500$ watts

Results:

Loss

$MtchdLoss_1 = 1.66$ dB

$TotalLoss_1 = 13.15$ dB

Reflection coefficient

$\left| \rho_{L1}(F_1) \right| = 0.978$

$\left| \rho_{IN1}(F_1) \right| = 0.667$

Standing wave ratio

$SWR_{L1}(F_1) = 90.37$

$SWR_{IN1}(F_1) = 5.01$

Input impedance

$Z_{IN1}(F_1) = 12.3719 - 25.6079i$ ohms

Power delivered to load

$P_{L1} = 72.7$ watts

Voltage stress

$V_{maxPIN1} = 619.4$ volts rms

Compare with rating:

$V_{max} = 1400$ volts rms

Power stress

$P_{TOT1perft} = 27.7$ watts/foot

TRANSMISSION LINE CALCULATIONS FOR ANY FREQUENCY, LENGTH AND INPUT POWER

WHEN THE INPUT IMPEDANCE IS KNOWN

Loss, SWR, load impedance and voltage and power stress for any frequency, length, input impedance and power input. The subscript "2" is used for this calculation. Modify the input data as desired.

Input data:

$F_2 := 14\ \text{MHz}$

$\text{Length}_2 := 100\ \text{feet}$

$Z_{IN2} := 12.3719 - 25.6079 \cdot j\ \text{ohms}$

$P_{IN2} := 1500\,\text{watts}$

Calculations:

$$Z_{L2}(f) := \frac{Z_{IN2} - Z_0(f)\cdot\tanh\left(\text{Length}_2\cdot\gamma(f)\right)}{1 - \dfrac{Z_{IN2}\cdot\tanh\left(\text{Length}_2\cdot\gamma(f)\right)}{Z_0(f)}}$$

Modified reflection coefficients at load and input:

$$\rho_{LM2}(f) := \frac{Z_{L2}(f) - \overline{Z_0(f)}}{Z_{L2}(f) + Z_0(f)}$$

$$\rho_{INM2}(f) := \frac{Z_{IN2} - \overline{Z_0(f)}}{Z_{IN2} + Z_0(f)}$$

Matched loss at F_2:

$$\text{MtchdLoss}_2 := \frac{\text{Length}_2}{100}\cdot A_{0TOT}\left(F_2\right)$$

Total loss at F_2:

$$\text{TotalLoss}_2 := \text{MtchdLoss}_2 + 10\cdot\log\left[\frac{1 - \left(\left|\rho_{INM2}\left(F_2\right)\right|\right)^2}{1 - \left(\left|\rho_{LM2}\left(F_2\right)\right|\right)^2}\right]$$

Classical reflection coefficients and SWR at load and input:

$$\rho_{L2}(f) := \frac{Z_{L2}(f) - Z_0(f)}{Z_{L2}(f) + Z_0(f)} \qquad \rho_{IN2}(f) := \frac{Z_{IN2} - Z_0(f)}{Z_{IN2} + Z_0(f)}$$

$$\text{SWR}_{L2}(f) := \frac{1 + \left|\rho_{L2}(f)\right|}{1 - \left|\rho_{L2}(f)\right|} \qquad \text{SWR}_{IN2}(f) := \frac{1 + \left|\rho_{IN2}(f)\right|}{1 - \left|\rho_{IN2}(f)\right|}$$

WHEN THE INPUT IMPEDANCE IS KNOWN (Continued)

Voltage and power stress calculation. We calculate the stress when the input power is P_{IN2}. Break the cable into 100 equal length segments.

$m := 0 .. 100 \qquad x_m := m \cdot \frac{Length_2}{100}$

Assume initially that 1 ampere rms flows in load impedance

$V_2(x) := Z_{L2}(F_2)\cdot\cosh(\gamma(F_2)\cdot x) + Z_0(F_2)\cdot\sinh(\gamma(F_2)\cdot x)$

$I_2(x) := \cosh(\gamma(F_2)\cdot x) + \frac{Z_{L2}(F_2)}{Z_0(F_2)}\cdot\sinh(\gamma(F_2)\cdot x)$

$V_{2mag}(x) := \left|V_2(x)\right|$

$V_{2mag_m} := V_{2mag}(x_m)$

$P_{2COND}(x) := \left(\left|I_2(x)\right|\right)^2\cdot R(F_2)$

$P_{2INS}(x) := \left(\left|V_2(x)\right|\right)^2\cdot G(F_2)$

$P_{2TOT}(x) := P_{2COND}(x) + P_{2INS}(x)$

$P_{2TOT_m} := P_{2TOT}(x_m)$

Compute maximum voltage and watts per foot along line when input power equals P_{IN2}

$P_{L2} := P_{IN2}\cdot 10^{\frac{-TotalLoss_2}{10}}$

$R_{L2} := Re(Z_{L2}(F_2))$

$I_{L2} := \sqrt{\frac{P_{L2}}{R_{L2}}}$

$V_{maxPIN2} := max(V_{2mag})\cdot I_{L2}$

$P_{TOT2perft} := I_{L2}^{\,2}\cdot max(P_{2TOT})$

Check: The sum of the power in the cable segments plus the power in the load should equal the input power. This calculation will not be exact because of the finite number of cable segments.

$$P_{CABLE2} := I_{L1}^{\,2}\cdot\left[\frac{1}{2}\cdot\left(P_{2TOT_0} + P_{2TOT_{100}}\right) + \sum_{m=1}^{99} P_{2TOT_m}\right]\cdot\frac{Length_2}{100}$$

$P_{CABLE2} = 1427.519$ watts

$P_{Input2} := P_{CABLE2} + P_{L2} \qquad P_{Input2} = 1500.196$ watts

Compare with: $\qquad P_{IN2} = 1500$ watts

TRANSMISSION LINE CALCULATIONS FOR ANY FREQUENCY, LENGTH AND INPUT POWER

WHEN THE INPUT IMPEDANCE IS KNOWN

SUMMARY

Input data:

$F_2 = 14$ MHz

$Length_2 = 100$ feet

$Z_{IN2} = 12.3719 - 25.6079i$ ohms

$P_{IN2} = 1500$ watts

Results:

Loss

$MtchdLoss_2 = 1.66$ dB

$TotalLoss_2 = 13.15$ dB

Reflection coefficient

$\left|\rho_{L2}(F_2)\right| = 0.978$

$\left|\rho_{IN2}(F_2)\right| = 0.667$

Standing wave ratio

$SWR_{L2}(F_2) = 90.37$

$SWR_{IN2}(F_2) = 5.01$

Load impedance

$Z_{L2}(F_2) = 50 - 500i$ ohms

Power delivered to load

$P_{L2} = 72.7$ watts

Voltage stress

$V_{maxPIN2} = 619.4$ volts rms

Compare with rating:

$V_{max} = 1400$ volts rms

Power stress

$P_{TOT2perft} = 27.7$ watts/foot

SERIES RESONATOR DESIGN

The length is changed to be a quarter wavelength or a half wavelength at the design frequency. The subscript QOC means quarter-wave open-circuited. The subscript HSC means half-wave short-circuited.

$F_{SR} := 21\ \text{MHz}$

Open-circuited quarter-wavelength segment

$\text{Length}_{quarter}(F_{SR}) = 7.73$ feet

$$Z_{INQOC}(f) := \frac{Z_0(f)}{\tanh\left(\text{Length}_{quarter}(F_{SR})\cdot\gamma(f)\right)}$$

$Z_{INQOC}(F_{SR}) = 0.916 - 9.475i\cdot 10^{-3}$ ohms

$\left|Z_{INQOC}(F_{SR})\right| = 0.916$ ohms

$$X_{QOC}(f) := \frac{f}{2}\cdot\frac{d}{df}\text{Im}\left(Z_{INQOC}(f)\right)$$

$X_{QOC}(F_{SR}) = 39.26$ ohms

$$Q_{QOC}(f) := \frac{X_{QOC}(f)}{\text{Re}\left(Z_{INQOC}(f)\right)}$$

$Q_{QOC}(F_{SR}) = 42.85$

Short-circuited half-wavelength segment

$\text{Length}_{half}(F_{SR}) = 15.461$ feet

$$Z_{INHSC}(f) := Z_0(f)\cdot\tanh\left(\text{Length}_{half}(F_{SR})\cdot\gamma(f)\right)$$

$Z_{INHSC}(F_{SR}) = 1.832 - 0.019i$ ohms

$\left|Z_{INHSC}(F_{SR})\right| = 1.832$ ohms

$$X_{HSC}(f) := \frac{f}{2}\cdot\frac{d}{df}\text{Im}\left(Z_{INHSC}(f)\right)$$

$X_{HSC}(F_{SR}) = 78.44$ ohms

$$Q_{HSC}(f) := \frac{X_{HSC}(f)}{\text{Re}\left(Z_{INHSC}(f)\right)}$$

$Q_{HSC}(F_{SR}) = 42.82$

PARALLEL RESONATOR DESIGN

The length is changed to be a quarter wavelength or a half wavelength at the design frequency. The subscript HOC means half-wave open-circuited. The subscript QSC means quarter-wave short-circuited.

$F_{PR} := 21\ \text{MHz}$

Open-circuited half-wavelength segment

$Length_{half}(F_{PR}) = 15.461$ feet

$$Z_{INHOC}(f) := \frac{Z_0(f)}{\tanh\left(Length_{half}(F_{PR})\cdot\gamma(f)\right)}$$

$Z_{INHOC}(F_{PR}) = 1365.02 - 14.12i$ ohms

$\left|Z_{INHOC}(F_{PR})\right| = 1365$ ohms

$$B_{HOC}(f) := \frac{f}{2}\cdot\frac{d}{df}\mathrm{Im}\left(\frac{1}{Z_{INHOC}(f)}\right)$$

$B_{HOC}(F_{PR}) = 0.03137$ siemens

$$Q_{HOC}(f) := \frac{B_{HOC}(f)}{\mathrm{Re}\left(\frac{1}{Z_{INHOC}(f)}\right)}$$

$Q_{HOC}(F_{PR}) = 42.82$

Short-circuited quarter-wavelength segment

$Length_{quarter}(F_{PR}) = 7.73$ feet

$$Z_{INQSC}(f) := Z_0(f)\cdot\tanh\left(Length_{quarter}(F_{PR})\cdot\gamma(f)\right)$$

$Z_{INQSC}(F_{PR}) = 2729.11 - 28.22i$ ohms

$\left|Z_{INQSC}(F_{PR})\right| = 2729$ ohms

$$B_{QSC}(f) := \frac{f}{2}\cdot\frac{d}{df}\mathrm{Im}\left(\frac{1}{Z_{INQSC}(f)}\right)$$

$B_{QSC}(F_{PR}) = 0.0157$ siemens

$$Q_{QSC}(f) := \frac{B_{QSC}(f)}{\mathrm{Re}\left(\frac{1}{Z_{INQSC}(f)}\right)}$$

$Q_{QSC}(F_{PR}) = 42.85$

INDUCTIVE AND CAPACITIVE REACTANCE DESIGN

The lengths required to achieve the desired inductive or capacitive reactance at any frequency are calculated. The lengths are the shortest ones that will achieve the desired reactance. The subscript L applies to the inductive reactance and the subscript C applies to the capacitive reactance. The inductive reactance occurs when the far end is shorted and the capacitive reactance is achieved when the far end is open circuited. Substitute the desired frequency and reactance for the ones shown.

$F_{LC} := 21\ \text{MHz}$ $\qquad$ $X_{desired} := 100\ \text{ohms}$

Inductive reactance $\qquad$ Shorted segment

$$Length_L := \frac{\operatorname{atan}\left[\dfrac{X_{desired}}{\left(R_0\left(F_{LC}\right)\right)}\right]}{\beta\left(F_{LC}\right)}$$

$Length_L = 5.45$ feet

$$Z_L := Z_0\left(F_{LC}\right)\cdot\tanh\left(Length_L\cdot\gamma\left(F_{LC}\right)\right)$$

$Z_L = 4.3 + 99.9i$ ohms

$$L_{effective} := \frac{\operatorname{Im}\left(Z_L\right)}{2\cdot\pi\cdot F_{LC}}$$

$L_{effective} = 0.757$ μH

$$Q_L := \frac{\operatorname{Im}\left(Z_L\right)}{\operatorname{Re}\left(Z_L\right)}$$

$Q_L = 23.4$

Capacitive reactance $\qquad$ Open-circuited segment

$$Length_C := \frac{\operatorname{atan}\left(\dfrac{R_0\left(F_{LC}\right)}{X_{desired}}\right)}{\beta\left(F_{LC}\right)}$$

$Length_C = 2.28$ feet

$$Z_C := \frac{Z_0\left(F_{LC}\right)}{\tanh\left(Length_C\cdot\gamma\left(F_{LC}\right)\right)}$$

$Z_C = 0.3 - 100i$ ohms

$$C_{effective} := \frac{\operatorname{Im}\left(\dfrac{1}{Z_C}\right)}{2\cdot\pi\cdot F_{LC}\cdot 10^{-6}}$$

$C_{effective} = 75.79$ pF

$$Q_C := \frac{-\operatorname{Im}\left(Z_C\right)}{\operatorname{Re}\left(Z_C\right)}$$

$Q_C = 314.5$