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# Smith Chart Calculations

The Smith Chart is a sophisticated graphic tool for solving transmission line problems. One of the simpler applications is to determine the feed-point impedance of an antenna, based on an impedance measurement at the input of a random length of transmission line. By using the Smith Chart, the impedance measurement can be made with the antenna in place atop a tower or mast, and there is no need to cut the line to an exact multiple of half wavelengths. The Smith Chart may be used for other purposes, too, such as the design of impedance-matching networks. These matching networks can take on any of several forms, such as L and pi networks, a stub matching system, a series-section match, and more. With a knowledge of the Smith Chart, the amateur can eliminate much “cut and try” work.

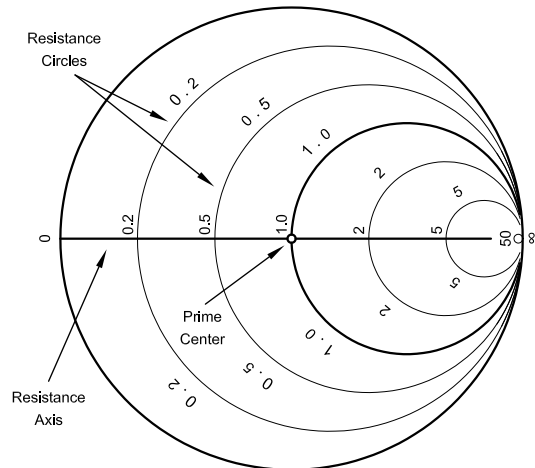
Named after its inventor, Phillip H. Smith, the Smith Chart was originally described in *Electronics* for January 1939. Smith Charts may be obtained at most university book stores. Smith Charts are also available from ARRL HQ. (See the caption for Fig 3.)

The input impedance, or the impedance seen when “looking into” a length of line, is dependent upon the SWR, the length of the line, and the  $Z_0$  of the line. The SWR, in turn, is dependent upon the load which terminates the line. There are complex mathematical relationships which may be used to calculate the various values of impedances, voltages, currents, and SWR values that exist in the operation of a particular transmission line. These equations can be solved with a personal computer and suitable software, or the parameters may be determined with the Smith Chart. Even if a computer is used, a fundamental knowledge of the Smith Chart will promote a better understanding of the problem being solved. And such an understanding might lead to a quicker or simpler solution than otherwise. If the terminating impedance is known, it is a simple matter to determine the input impedance of the line for any length by means of the chart. Conversely, as indicated above, with a given line length and a known (or measured) input impedance, the load impedance may be determined by means of the chart—a convenient method of remotely determining an antenna impedance, for example.

Although its appearance may at first seem somewhat formidable, the Smith Chart is really nothing more than a

specialized type of graph. Consider it as having curved, rather than rectangular, coordinate lines. The coordinate system consists simply of two families of circles—the resistance family, and the reactance family. The resistance circles, **Fig 1**, are centered on the resistance axis (the only straight line on the chart), and are tangent to the outer circle at the right of the chart. Each circle is assigned a value of resistance, which is indicated at the point where the circle crosses the resistance axis. All points along any one circle have the same resistance value.

The values assigned to these circles vary from zero at the left of the chart to infinity at the right, and actually represent a *ratio* with respect to the impedance value assigned to the center point of the chart, indicated 1.0. This center point is called prime center. If prime center is assigned a value of 100  $\Omega$ , then 200  $\Omega$  resistance is represented by the 2.0 circle, 50  $\Omega$  by the 0.5 circle, 20  $\Omega$  by the 0.2 circle, and so on. If, instead, a value of 50 is assigned to prime center, the 2.0 circle now represents 100  $\Omega$ , the 0.5 circle 25  $\Omega$ , and the 0.2 circle 10  $\Omega$ . In each case, it may be seen that the value on the chart is determined by dividing the actual resistance by the number



**Fig 1—Resistance circles of the Smith Chart coordinate system.**

assigned to prime center. This process is called normalizing.

Conversely, values from the chart are converted back to actual resistance values by multiplying the chart value times the value assigned to prime center. This feature permits the use of the Smith Chart for any impedance values, and therefore with any type of uniform transmission line, whatever its impedance may be. As mentioned above, specialized versions of the Smith Chart may be obtained with a value of 50 Ω at prime center. These are intended for use with 50-Ω lines.

Now consider the reactance circles, **Fig 2**, which appear as curved lines on the chart because only segments of the complete circles are drawn. These circles are tangent to the resistance axis, which itself is a member of the reactance family (with a radius of infinity). The centers are displaced to the top or bottom on a line tangent to the right of the chart. The large outer circle bounding the coordinate portion of the chart is the reactance axis.

Each reactance circle segment is assigned a value of reactance, indicated near the point where the circle touches the reactance axis. All points along any one segment have the same reactance value. As with the resistance circles, the values assigned to each reactance circle are normalized with respect to the value assigned to prime center. Values to the top of the resistance axis are positive (inductive), and those to the bottom of the resistance axis are negative (capacitive).

When the resistance family and the reactance family of circles are combined, the coordinate system of the Smith Chart results, as shown in **Fig 3**. Complex impedances ( $R + jX$ ) can be plotted on this coordinate system.

## IMPEDANCE PLOTTING

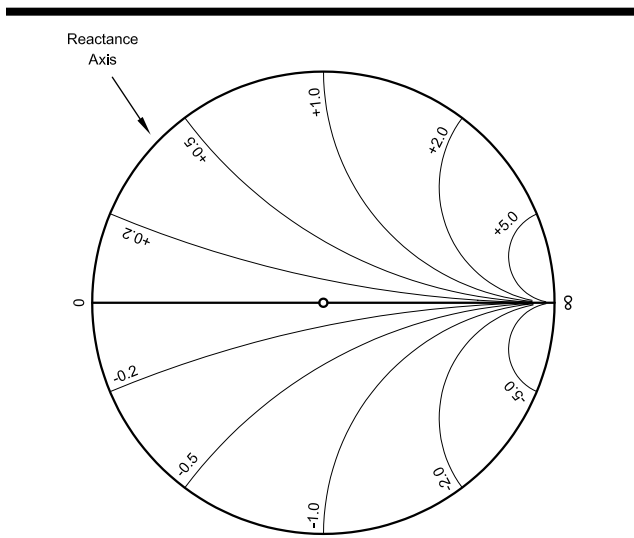
Suppose we have an impedance consisting of 50 Ω resistance and 100 Ω inductive reactance ( $Z = 50 + j 100$ ). If we assign a value of 100 Ω to prime center, we normalize the above impedance by dividing each component of the

impedance by 100. The normalized impedance is then  $50/100 + j(100/100) = 0.5 + j 1.0$ . This impedance is plotted on the Smith Chart at the intersection of the 0.5 resistance circle and the +1.0 reactance circle, as indicated in **Fig 3**. Calculations may now be made from this plotted value.

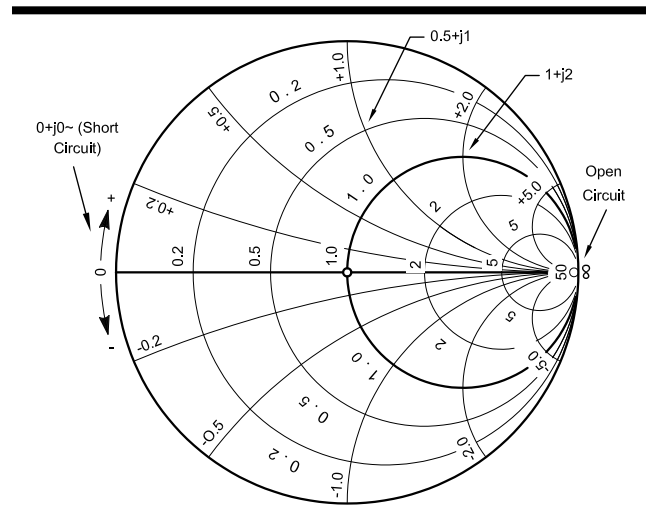
Now say that instead of assigning 100 Ω to prime center, we assign a value of 50 Ω. With this assignment, the  $50 + j 100$  Ω impedance is plotted at the intersection of the  $50/50 = 1.0$  resistance circle, and the  $100/50 = 2.0$  positive reactance circle. This value,  $1 + j 2$ , is also indicated in **Fig 3**. But now we have *two* points plotted in **Fig 3** to represent the same impedance value,  $50 + j 100$  Ω. How can this be?

These examples show that the same impedance may be plotted at different points on the chart, depending upon the value assigned to prime center. But two plotted points cannot represent the same impedance *at the same time*! It is customary when solving transmission-line problems to assign to prime center a value equal to the characteristic impedance, or  $Z_0$ , of the line being used. This value should always be recorded at the start of calculations, to avoid possible confusion later. (In using the specialized charts with the value of 50 at prime center, it is, of course, not necessary to normalize impedances when working with 50-Ω line. The resistance and reactance values may be read directly from the chart coordinate system.)

Prime center is a point of special significance. As just mentioned, it is customary when solving problems to assign the  $Z_0$  value of the line to this point on the chart—50 Ω for a 50-Ω line, for example. What this means is that the center point of the chart now represents  $50 + j 0$  ohms—a pure resistance equal to the characteristic impedance of the line. If this were a load on the line, we recognize from transmission-line theory that it represents a perfect match, with no reflected



**Fig 2—Reactance circles (segments) of the Smith Chart coordinate system.**



**Fig 3—The complete coordinate system of the Smith Chart. For simplicity, only a few divisions are shown for the resistance and reactance values. Various types of Smith Chart forms are available from ARRL HQ. At the time of this writing, five 8 1/2 × 11 inch Smith Chart forms are available for \$2.**

power and with a 1.0 to 1 SWR. Thus, prime center also represents the 1.0 SWR circle (with a radius of zero). SWR circles are also discussed in a later section.

### Short and Open Circuits

On the subject of plotting impedances, two special cases deserve consideration. These are short circuits and open circuits. A true short circuit has zero resistance and zero reactance, or  $0 + j0$ . This impedance is plotted at the left of the chart, at the intersection of the resistance and the reactance axes. By contrast, an open circuit has infinite resistance, and therefore is plotted at the right of the chart, at the intersection of the resistance and reactance axes. These two special cases are sometimes used in matching stubs, described later.

### Standing-Wave-Ratio Circles

Members of a third family of circles, which are not printed on the chart but which are added during the process of solving problems, are standing-wave-ratio or SWR circles. See Fig 4. This family is centered on prime center, and appears as concentric circles inside the reactance axis. During calculations, one or more of these circles may be added with a drawing compass. Each circle represents a value of SWR, with every point on a given circle representing the same SWR. The SWR value for a given circle may be determined directly from the chart coordinate system, by reading the resistance value where the SWR circle crosses the resistance axis to the right of prime center. (The reading where the circle crosses the resistance axis to the left of prime center indicates the inverse ratio.)

Consider the situation where a load mismatch in a length of line causes a 3-to-1 SWR ratio to exist. If we temporarily disregard line losses, we may state that the SWR remains constant throughout the entire length of this line. This is represented on the Smith Chart by drawing a 3:1 constant SWR circle (a circle with a radius of 3 on the resistance axis), as in Fig 5. The design of the chart is such that any impedance encountered *anywhere* along the length of this mismatched line will fall on the SWR circle. The impedances may be read from the coordinate system merely by the progressing around the SWR circle by an amount corresponding to the length of the line involved.

This brings into use the wavelength scales, which appear in Fig 5 near the perimeter of the Smith Chart. These scales are calibrated in terms of portions of an electrical wavelength along a transmission line. Both scales start from 0 at the left of the chart. One scale, running counterclockwise, starts at the generator or input end of the line and progresses toward the load. The other scale starts at the load and proceeds toward the generator in a clockwise direction. The complete circle around the edge of the chart represents  $1/2 \lambda$ . Progressing once around the perimeter of these scales cor-

responds to progressing along a transmission line for  $1/2 \lambda$ . Because impedances repeat themselves every  $1/2 \lambda$  along a piece of line, the chart may be used for any length of line by disregarding or subtracting from the line's total length an integral, or whole number, of half wavelengths.

Also shown in Fig 5 is a means of transferring the

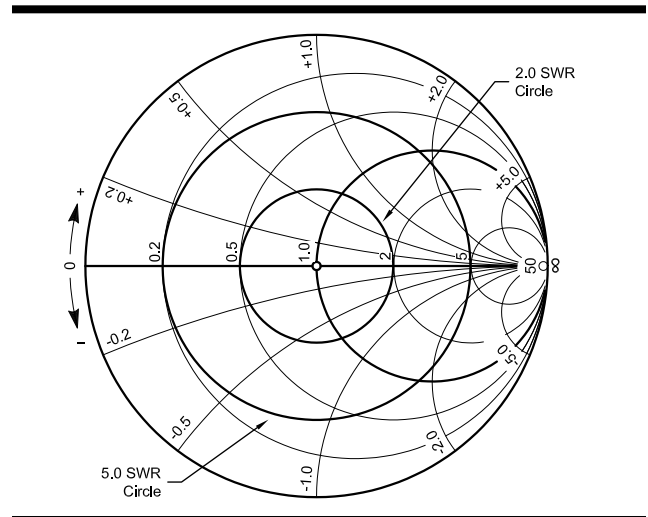


Fig 4—Smith Chart with SWR circles added.

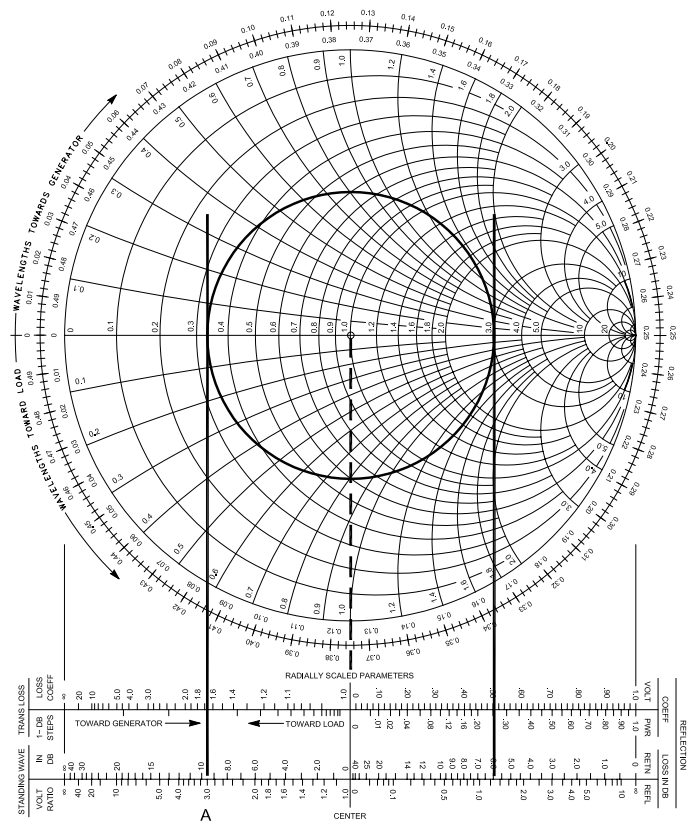


Fig 5—Example discussed in text.

radius of the SWR circle to the external scales of the chart, by drawing lines tangent to the circle. Another simple way to obtain information from these external scales is to transfer the radius of the SWR circle to the external scale with a drawing compass. Place the point of a drawing compass at the center or 0 line, and inscribe a short arc across the appropriate scale. It will be noted that when this is done in Fig 5, the external STANDING-WAVE VOLTAGE-RATIO scale indicates the SWR to be 3.0 (at A)—our condition for initially drawing the circle on the chart (and the same as the SWR reading on the resistance axis).

## SOLVING PROBLEMS WITH THE SMITH CHART

Suppose we have a transmission line with a characteristic impedance of  $50\ \Omega$  and an electrical length of  $0.3\ \lambda$ . Also, suppose we terminate this line with an impedance having a resistive component of  $25\ \Omega$  and an inductive reactance of  $25\ \Omega$  ( $Z = 25 + j25$ ). What is the input impedance to the line?

The characteristic impedance of the line is  $50\ \Omega$ , so we begin by assigning this value to prime center. Because the line is not terminated in its characteristic impedance, we know that standing waves will exist on the line, and that, therefore, the input impedance to the line will not be exactly  $50\ \Omega$ . We proceed as follows. First, normalize the load impedance by dividing both the resistive and reactive components by  $50$  ( $Z_0$  of the line being used). The normalized impedance in this case is  $0.5 + j0.5$ . This is plotted on the chart at the intersection of the  $0.5$  resistance and the  $+0.5$  reactance circles, as in Fig 6. Then draw a constant SWR circle passing through this point. Transfer the radius of this circle to the external scales with the drawing compass. From the external STANDING-WAVE VOLTAGE-RATIO scale, it may be seen (at A) that the voltage ratio of 2.62 exists for this radius, indicating that our line is operating with an SWR of 2.62 to 1. This figure is converted to decibels in the adjacent scale, where 8.4 dB may be read (at B), indicating that the ratio of the voltage maximum to the voltage minimum along the line is 8.4 dB. (This is mathematically equivalent to 20 times the log of the SWR value.)

Next, with a straightedge, draw a radial line from prime center through the plotted point to intersect the wavelengths scale. At this intersection, point C in Fig 6, read a value from the wavelengths scale. Because we are starting from the load, we use the TOWARD GENERATOR or outermost calibration, and read  $0.088\ \lambda$ .

To obtain the line input impedance, we merely find the point on the SWR circle that is  $0.3\ \lambda$  toward the generator from the plotted load impedance. This is accomplished by adding  $0.3$  (the length of the line in wavelengths) to the reference or starting point,  $0.088$ ;  $0.3 + 0.088 = 0.388$ . Locate  $0.388$  on the TOWARD GENERATOR scale (at D). Draw a second radial line from this point to prime center.

The intersection of the new radial line with the SWR circle represents the normalized line input impedance, in this case  $0.6 - j0.66$ .

To find the unnormalized line impedance, multiply by  $50$ , the value assigned to prime center. The resulting value is  $30 - j33$ , or  $30\ \Omega$  resistance and  $33\ \Omega$  capacitive reactance. This is the impedance that a transmitter must match if such a system were a combination of antenna and transmission line. This is also the impedance that would be measured on an impedance bridge if the measurement were taken at the line input.

In addition to the line input impedance and the SWR, the chart reveals several other operating characteristics of the above system of line and load, if a closer look is desired. For example, the voltage reflection coefficient, both magnitude and phase angle, for this particular load is given. The phase angle is read under the radial line drawn through the plot of the load impedance, where the line intersects the ANGLE OF REFLECTION COEFFICIENT scale. This scale is not included in Fig 6, but will be found on the Smith Chart just inside the wavelengths scales. In this example, the reading is  $116.6$  degrees. This indicates the angle by which the reflected voltage wave leads the incident wave at the load. It will be noted that angles on the bottom half, or capacitive-reactance half,

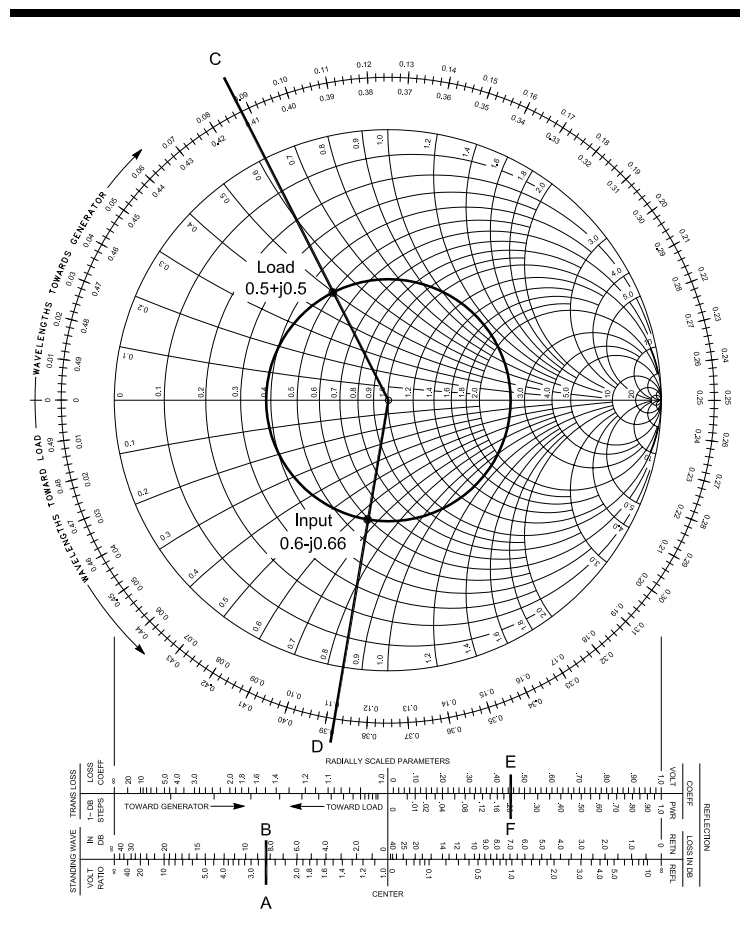


Fig 6—Example discussed in text.



case the value is 0.195, on the TOWARD LOAD scale. Remember, we are starting at the generator end of the transmission line.

To locate the load impedance on the SWR circle, add the line length,  $2.35 \lambda$ , to the reference value from the wavelengths scale;  $2.35 + 0.195 = 2.545$ . Locate the new value on the TOWARD LOAD scale. But because the calibrations extend only from 0 to 0.5, we must first subtract a number of half wavelengths from this value and use only the remaining value. In this situation, the largest integral number of half wavelengths that can be subtracted with a positive result is 5, or  $2.5 \lambda$ . Thus,  $2.545 - 2.5 = 0.045$ . Locate the 0.045 value on the TOWARD LOAD scale (at C). Draw a radial line from this value to prime center. Now, the coordinates at the intersection of the second radial line and the SWR circle represent the load impedance. To read this value closely, some interpolation between the printed coordinate lines must be made, and the value of  $0.62 - j 0.19$  is read. Multiplying by 50, we get the actual load or antenna impedance as  $31 - j 9.5 \Omega$ , or  $31 \Omega$  resistance with  $9.5 \Omega$  capacitive reactance.

Problems may be entered on the chart in yet another manner. Suppose we have a length of  $50\text{-}\Omega$  line feeding a base-loaded resonant vertical ground-plane antenna which is shorter than  $\frac{1}{4} \lambda$ . Further, suppose we have an SWR monitor in the line, and that it indicates an SWR of 1.7 to 1. The line is known to be  $0.95 \lambda$  long. We want to know both the input and the antenna impedances.

From the information available, we have no impedances to enter into the chart. We may, however, draw a circle representing the 1.7 SWR. We also know, from the definition of resonance, that the antenna presents a purely resistive load to the line, that is, no reactive component. Thus, the antenna impedance must lie on the resistance axis. If we were to draw such an SWR circle and observe the chart with only the circle drawn, we would see two points which satisfy the resonance requirement for the load. These points are  $0.59 + j 0$  and  $1.7 + j 0$ . Multiplying by 50, we see that these values represent 29.5 and  $85 \Omega$  resistance. This may sound familiar, because, when a line is terminated in a pure resistance, the SWR in the line equals  $Z_R/Z_0$  or  $Z_0/Z_R$ , where  $Z_R$ =load resistance and  $Z_0$ =line impedance.

If we consider antenna fundamentals, we know that the theoretical impedance of a  $\frac{1}{4}\text{-}\lambda$  ground-plane antenna is approximately  $36 \Omega$ . We therefore can quite logically discard the  $85\text{-}\Omega$  impedance figure in favor of the  $29.5\text{-}\Omega$  value. This is then taken as the load impedance value for the Smith Chart calculations. To find the line input impedance, we subtract  $0.5 \lambda$  from the line length, 0.95, and find  $0.45 \lambda$  on the TOWARD GENERATOR scale. (The wavelength-scale starting point in this case is 0.) The line input impedance is found to be  $0.63 - j 0.20$ , or  $31.5 - j 10 \Omega$ .

## DETERMINATION OF LINE LENGTH

In the example problems given so far in this chapter, the line length has conveniently been stated in wavelengths. The electrical length of a piece of line depends upon its physical length, the radio frequency under consideration, and the velocity of propagation in the line. If an impedance-measurement bridge is capable of quite reliable readings at high SWR values, the line length may be determined through line input-impedance measurements with short- or open-circuit line terminations. Information on the procedure is given later in this chapter. A more direct method is to measure the physical length of the line and calculate its electrical length from

$$N = \frac{f L}{984 VF} \quad (\text{Eq 1})$$

where

N = number of electrical wavelengths in the line

L = line length in feet

f = frequency, MHz

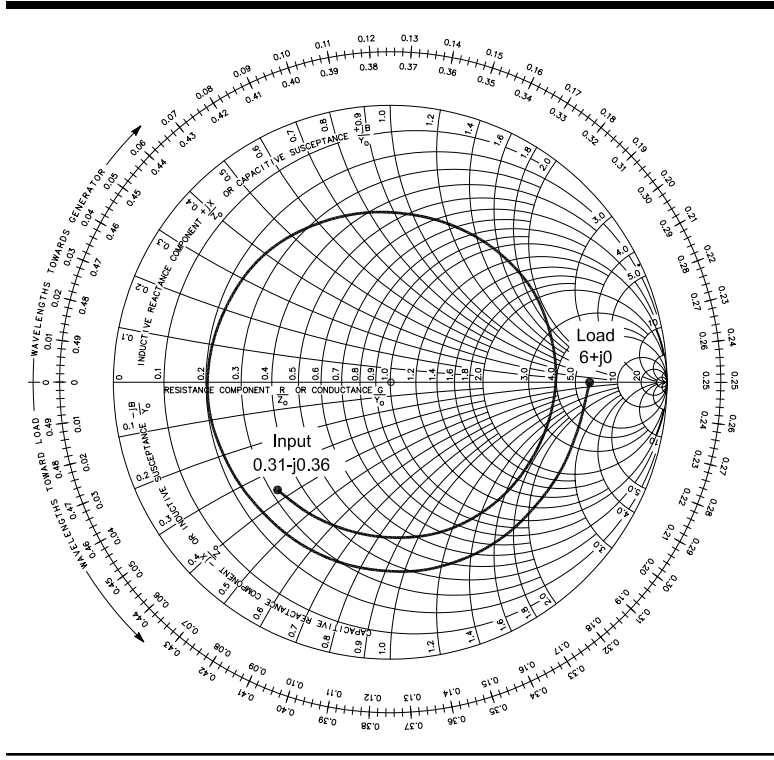
VF = velocity or propagation factor of the line

The velocity factor may be obtained from transmission-line data tables.

## LINE-LOSS CONSIDERATIONS WITH THE SMITH CHART

The example Smith Chart problems presented in the previous section ignored attenuation, or line losses. Quite frequently it is not even necessary to consider losses when making calculations; any difference in readings obtained are often imperceptible on the chart. However, when the line losses become appreciable, such as for high-loss lines, long lines, or at VHF and UHF, loss considerations may become significant in making Smith Chart calculations. This involves only one simple step, in addition to the procedures previously presented.

Because of line losses, the SWR does not remain constant throughout the length of the line. As a result, there is a decrease in SWR as one progresses away from the load. To truly present this situation on the Smith Chart, instead of drawing a constant SWR circle, it would be necessary to draw a spiral inward and clockwise from the load impedance toward the generator, as shown in **Fig 8**. The rate at which the curve spirals toward prime center is related to the attenuation in the line. Rather than drawing spiral curves, a simpler method is used in solving line-loss problems, by means of the external scale TRANSMISSION LOSS 1-DB STEPS. This scale may be seen in **Fig 9**. Because this is only a relative scale, the decibel steps are not numbered.



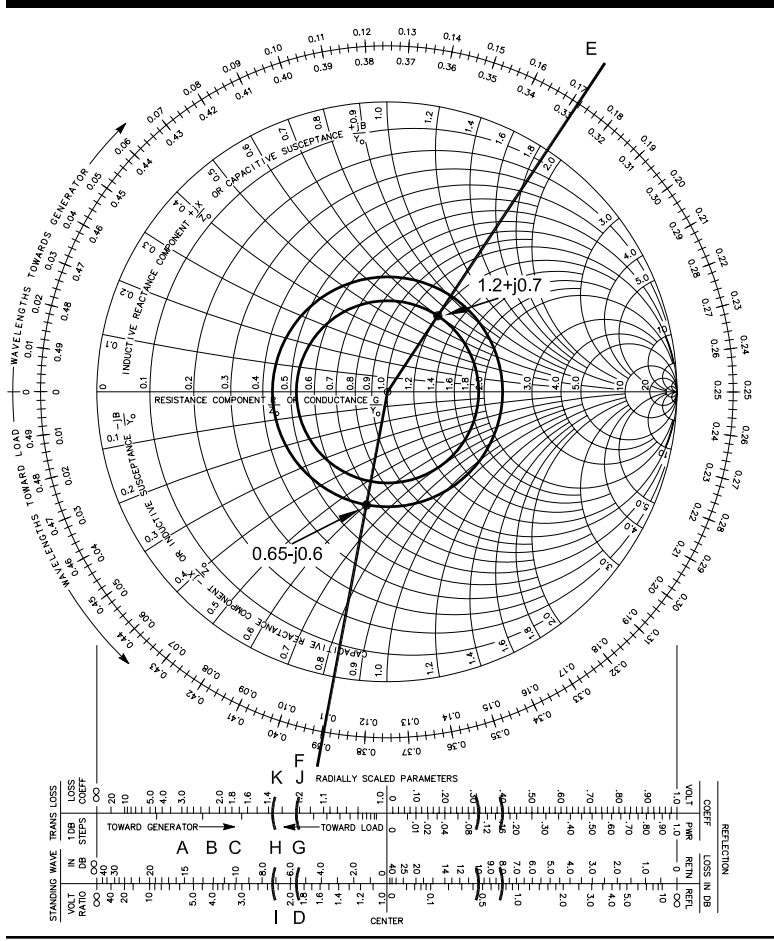
**Fig 8**—This spiral is the actual “SWR circle” when line losses are taken into account. It is based on calculations for a 16-ft length of RG-174 coax feeding a resonant 28-MHz 300-Ω antenna (50-Ω coax, velocity factor = 66%, attenuation = 6.2 dB per 100 ft). The SWR at the load is 6:1, while it is 3.6:1 at the line input. When solving problems involving attenuation, two constant SWR circles are drawn instead of a spiral, one for the line input SWR and one for the load SWR.

If we start at the left end of this external TRANSMISSION-LOSS 1-DB STEPS scale and proceed in the direction indicated TOWARD GENERATOR, the first dB step is seen to occur at a radius from center corresponding to an SWR of about 9 (at A); the second dB step falls at an SWR of about 4.5 (at B), the third at 3.0 (at C), and so forth, until the 15th dB step falls at an SWR of about 1.05 to 1. This means that a line terminated in a short or open circuit (infinite SWR), and having an attenuation of 15 dB, would exhibit an SWR of only 1.05 at its input. It will be noted that the dB steps near the right end of the scale are very close together, and a line attenuation of 1 or 2 dB in this area will have only slight effect on the SWR. But near the left end of the scale, corresponding to high SWR values, a 1 or 2 dB loss has considerable effect on the SWR.

**Using a Second SWR Circle**

In solving a problem using line-loss information, it is necessary only to modify the radius of the SWR circle by an amount indicated on the TRANSMISSION-LOSS 1-DB STEPS scale. This is accomplished by drawing a second SWR circle, either smaller or larger than the first, depending on whether you are working toward the load or toward the generator.

For example, assume that we have a 50-Ω line that is 0.282 λ long, with 1-dB inherent attenuation. The line input impedance is measured as 60 + j 35 Ω. We desire to know the SWR at the input and at the load, and the load impedance. As before, we normalize the 60 + j 35-Ω impedance, plot it on the chart, and draw a constant SWR circle and a radial line through the point. In this case, the normalized impedance is 1.2 + j 0.7. From Fig 9, the SWR at the line input is seen to be 1.9 (at D), and the radial line is seen to cross the TOWARD LOAD scale, first subtract 0.500, and locate 0.110 (at F); then draw a radial line from this point to prime center.



**Fig 9**—Example of Smith Chart calculations taking line losses into account.

To account for line losses, transfer the radius of the SWR circle to the external 1-DB STEPS scale. This radius crosses the external scale at G, the fifth decibel mark from the left. Since the line loss was given as 1 dB, we strike a new radius (at H), one “tick mark” to the left (toward load) on the same scale. (This will be the fourth decibel tick mark from the left of the scale.) Now transfer this new radius back to the main chart, and scribe a new SWR circle of this radius. This new radius represents the SWR at the load, and is read as 2.3 on the external VOLTAGE RATIO scale. At the intersection of the new circle and the load radial line, we read  $0.65 - j 0.6$ . This is the normalized load impedance. Multiplying by 50, we obtain the actual load impedance as  $32.5 - j 30 \Omega$ . The SWR in this problem was seen to increase from 1.9 at the line input to 2.3 (at I) at the load, with the 1-dB line loss taken into consideration.

In the example above, values were chosen to fall conveniently on or very near the “tick marks” on the 1-dB scale. Actually, it is a simple matter to interpolate between these marks when making a radius correction. When this is necessary, the relative distance between marks for each decibel step should be maintained while counting off the proper number of steps.

Adjacent to the 1-DB STEPS scale lies a LOSS COEFFICIENT scale. This scale provides a factor by which the matched-line loss in decibels should be multiplied to account for the increased losses in the line when standing waves are present. These added losses do not affect the SWR or impedance calculations; they are merely the additional dielectric and copper losses caused by the higher voltages and currents in the presence of standing waves. For the above example, from Fig 9, the loss coefficient at the input end is seen to be 1.21 (at J), and 1.39 (at K) at the load. As a good approximation, the loss coefficient may be averaged over the length of line under consideration; in this case, the average is 1.3. This means that the total losses in the line are 1.3 times the matched loss of the line (1 dB), or 1.3 dB.

### Smith Chart Procedure Summary

To summarize briefly, any calculations made on the Smith Chart are performed in four basic steps, although not necessarily in the order listed.

- 1) Normalize and plot a line input (or load) impedance, and construct a constant SWR circle.
- 2) Apply the line length to the wavelengths scales.
- 3) Determine attenuation or loss, if required, by means of a second SWR circle.
- 4) Read normalized load (or input) impedance, and convert to impedance in ohms.

The Smith Chart may be used for many types of problems other than those presented as examples here. The transformer action of a length of line—to transform a high impedance (with perhaps high reactance) to a purely resistive impedance of low value—was not mentioned. This is known as “tuning the line,” for which the chart is very helpful, eliminating the need for “cut and try” procedures. The chart may

also be used to calculate lengths for shorted or open matching stubs in a system, described later in this chapter. In fact, in any application where a transmission line is not perfectly matched, the Smith Chart can be of value.

## ATTENUATION AND $Z_0$ FROM IMPEDANCE MEASUREMENTS

If an impedance bridge is available to make accurate measurements in the presence of very high SWR values, the attenuation, characteristic impedance and velocity factor of any random length of coaxial transmission line can be determined. This section was written by Jerry Hall, K1TD.

Homemade impedance bridges and noise bridges will seldom offer the degree of accuracy required to use this technique, but sometimes laboratory bridges can be found as industrial surplus at a reasonable price. It may also be possible for an amateur to borrow a laboratory type of bridge for the purpose of making some weekend measurements. Making these determinations is not difficult, but the procedure is not commonly known among amateurs. One equation treating complex numbers is used, but the math can be handled with a calculator supporting trig functions. Full details are given in the paragraphs that follow.

For each frequency of interest, two measurements are required to determine the line impedance. Just one measurement is used to determine the line attenuation and velocity factor. As an example, assume we have a 100-foot length of unidentified line with foamed dielectric, and wish to know its characteristics. We make our measurements at 7.15 MHz. The procedure is as follows.

- 1) Terminate the line in an open circuit. The best “open circuit” is one that minimizes the capacitance between the center conductor and the shield. If the cable has a PL-259 connector, unscrew the shell and slide it back down the coax for a few inches. If the jacket and insulation have been removed from the end, fold the braid back along the outside of the line, away from the center conductor.
- 2) Measure and record the impedance at the input end of the line. If the bridge measures admittance, convert the measured values to resistance and reactance. Label the values as  $R_{oc} + j X_{oc}$ . For our example, assume we measure  $85 + j179 \Omega$ . (If the reactance term is capacitive, record it as negative.)
- 3) Now terminate the line in a short circuit. If a connector exists at the far end of the line, a simple short is a mating connector with a very short piece of heavy wire soldered between the center pin and the body. If the coax has no connector, removing the jacket and center insulation from a half inch or so at the end will allow you to tightly twist the braid around the center conductor. A small clamp or alligator clip around the outer braid at the twist will keep it tight.
- 4) Again measure and record the impedance at the input end of the line. This time label the values as  $R_{sc} \pm j X$ . Assume the measured value now is  $4.8 - j 11.2 \Omega$ .



This completes the measurements. Now we reach for the calculator.

As amateurs we normally assume that the characteristic impedance of a line is purely resistive, but it can (and does) have a small capacitive reactance component. Thus, the  $Z_0$  of a line actually consists of  $R_0 + j X_0$ . The basic equation for calculating the characteristic impedance is

$$Z_0 = \sqrt{Z_{oc} \times Z_{sc}} \quad (\text{Eq 2})$$

where

$$Z_{oc} = R_{oc} + jX_{oc}$$

$$Z_{sc} = R_{sc} + jX_{sc}$$

From Eq 2 the following working equation may be derived.

$$Z_0 = \sqrt{(R_{oc}R_{sc} - X_{oc}X_{sc}) + j(R_{oc}X_{sc} + R_{sc}X_{oc})} \quad (\text{Eq 3})$$

The expression under the radical sign in Eq 3 is in the form of  $R + j X$ . By substituting the values from our example into Eq 3, the R term becomes  $85 \times 4.8 - 179 \times (-11.2) = 2412.8$ , and the X term becomes  $85 \times (-11.2) + 4.8 \times 179 = -92.8$ . So far, we have determined that

$$Z_0 = \sqrt{2412.8 - j92.8\Omega}$$

The quantity under the radical sign is in rectangular form. Extracting the square root of a complex term is handled easily if it is in polar form, a vector value and its angle. The vector value is simply the square root of the sum of the squares, which in this case is

$$\sqrt{2412.8^2 + 92.8^2} = \sqrt{2414.58}$$

The tangent of the vector angle we are seeking is the value of the reactance term divided by the value of the resistance term. For our example this is  $\arctan -92.8/2412.8 = \arctan -0.03846$ . The angle is thus found to be  $-2.20^\circ$ . From all of this we have determined that

$$Z_0 = \sqrt{2414.58} \angle -2.20^\circ$$

Extracting the square root is now simply a matter of finding the square root of the vector value, and taking half the angle. (The angle is treated mathematically as an exponent.)

Our result for this example is  $Z_0 = 49.1 \angle -1.1^\circ$ . The small negative angle may be ignored, and we now know that we have coax with a nominal 50- $\Omega$  impedance. (Departures of as much as 6 to 8% from the nominal value are not uncommon.) If the negative angle is large, or if the angle is positive, you should recheck your calculations and perhaps even recheck the original measurements. You can get an idea of the validity of the measurements by normalizing the measured values to the calculated impedance and plotting them on a Smith Chart as shown in **Fig 10** for this example. Ideally, the two points should be diametrically opposite,

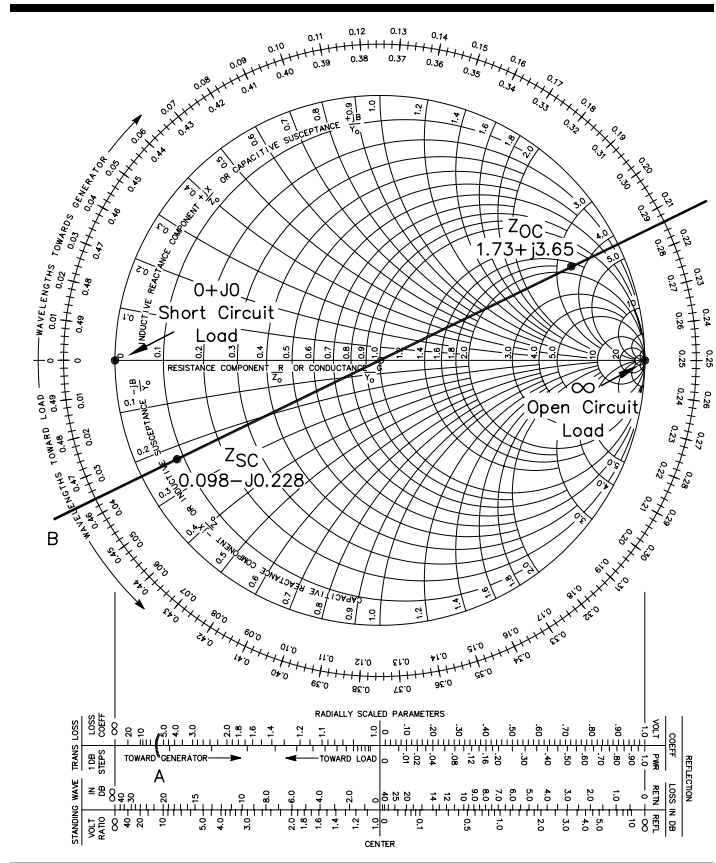
but in practice they will be not quite  $180^\circ$  apart and not quite the same distance from prime center. Careful measurements will yield plotted points that are close to ideal. Significant departures from the ideal indicates sloppy measurements, or perhaps an impedance bridge that is not up to the task.

### Determining Line Attenuation

The short circuit measurement may be used to determine the line attenuation. This reading is more reliable than the open circuit measurement because a good short circuit is a short, while a good open circuit is hard to find. (It is impossible to escape some amount of capacitance between conductors with an “open” circuit, and that capacitance presents a path for current to flow at the RF measurement frequency.)

Use the Smith Chart and the 1-DB STEPS external scale to find line attenuation. First normalize the short circuit impedance reading to the calculated  $Z_0$ , and plot this point on the chart. See Fig 10. For our example, the normalized impedance is  $4.8/49.1 - j 11.2 / 49.1$  or  $0.098 - j 0.228$ . After plotting the point, transfer the radius to the 1-DB STEPS scale. This is shown at A of Fig 10.

Remember from discussions earlier in this document



**Fig 10—Determining the line loss and velocity factor with the Smith Chart from input measurements taken with open-circuit and short-circuit terminations.**

that the impedance for plotting a short circuit is  $0 + j 0$ , at the left edge of the chart on the resistance axis. On the 1-DB STEPS scale this is also at the left edge. The total attenuation in the line is represented by the number of dB steps from the left edge to the radius mark we have just transferred. For this example it is 0.8 dB. Some estimation may be required in interpolating between the 1-dB step marks.

### Determining Velocity Factor

The velocity factor is determined by using the TOWARD GENERATOR wavelength scale of the Smith Chart. With a straightedge, draw a line from prime center through the point representing the short-circuit reading, until it intersects the wavelengths scale. In Fig 10 this point is labeled B. Consider that during our measurement, the short circuit was the load at the end of the line. Imagine a spiral curve progressing from  $0 + j 0$  clockwise and inward to our plotted measurement point. The wavelength scale, at B, indicates this line length is  $0.464 \lambda$ . By rearranging the terms of Eq 1 given early in this chapter, we arrive at an equation for calculating the velocity factor.

$$VF = \frac{f L}{984N} \quad (\text{Eq 4})$$

where

- VF = velocity factor
- L = line length, feet
- f = frequency, MHz
- N = number of electrical wavelengths in the line

Inserting the example values into Eq 4 yields  $VF = 100 \times 7.15 / (984 \times 0.464) = 1.566$ , or 156.6%. Of course, this value is an impossible number—the velocity factor in coax cannot be greater than 100%. But remember, the Smith Chart can be used for lengths greater than  $1/2 \lambda$ . Therefore, that 0.464 value could rightly be 0.964, 1.464, 1.964, and so on. When using  $0.964 \lambda$ , Eq 4 yields a velocity factor of 0.753, or 75.3%. Trying successively greater values for the wavelength results in velocity factors of 49.6 and 37.0%. Because the cable we measured had foamed dielectric, 75.3% is the probable velocity factor. This corresponds to an electrical length of  $0.964 \lambda$ . Therefore, we have determined from the measurements and calculations that our unmarked coax has a nominal  $50\text{-}\Omega$  impedance, an attenuation of 0.8 dB per hundred feet at 7.15 MHz, and a velocity factor of 75.3%.

It is difficult to use this procedure with short lengths of coax, just a few feet. The reason is that the SWR at the line input is too high to permit accurate measurements with most impedance bridges. In the example above, the SWR at the line input is approximately 12:1.

The procedure described above may also be used for determining the characteristics of balanced lines. However, impedance bridges are generally unbalanced devices, and the procedure for measuring a balanced impedance accurately with an unbalanced bridge is complicated.

## LINES AS CIRCUIT ELEMENTS

Transmission-line sections may also be used as circuit elements. For example, it is possible to substitute transmission lines of the proper length and termination for coils or capacitors in ordinary circuits. While there is seldom a practical need for that application, lines are frequently used in antenna systems in place of lumped components to tune or resonate elements. Probably the most common use of such a line is in the hairpin match, where a short section of stiff open-wire line acts as a lumped inductor.

The equivalent “lumped” value for any “inductor” or “capacitor” may be determined with the aid of the Smith Chart. Line losses may be taken into account if desired, as explained earlier. See Fig 11. Remember that the top half of the Smith Chart coordinate system is used for impedances containing inductive reactances, and the bottom half for capacitive reactances. For example, a section of  $600\text{-}\Omega$  line  $3/16\text{-}\lambda$  long ( $0.1875 \lambda$ ) and short-circuited at the far end is represented by  $\ell 1$ , drawn around a portion of the perimeter of the chart. The “load” is a short-circuit,  $0 + j 0 \Omega$ , and the TOWARD GENERATOR wavelengths scale is used for marking off the line length. At A in

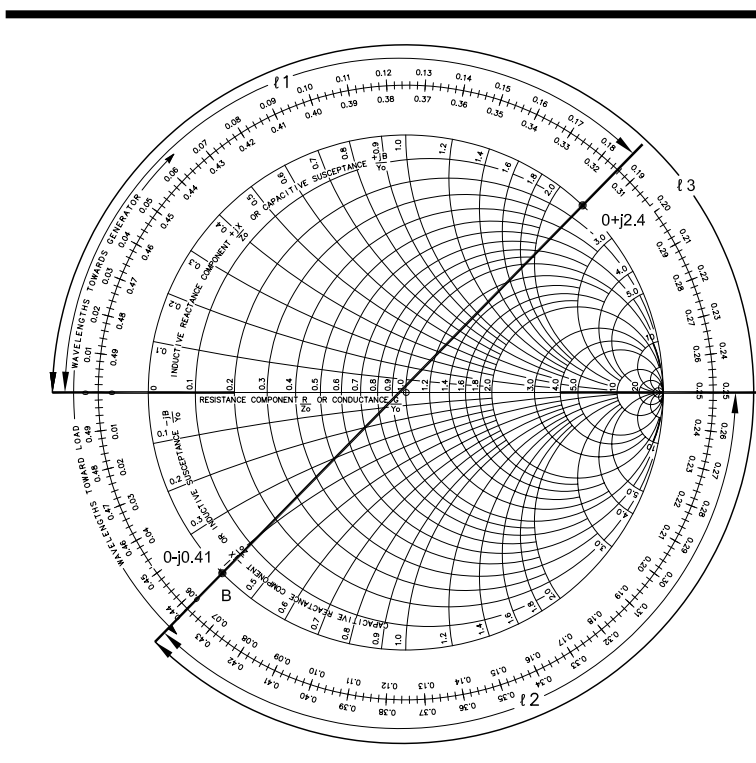


Fig 11—Smith Chart determination of input impedances for short- and open-circuited line sections, disregarding line losses.

Fig 11 may be read the normalized impedance as seen looking into the length of line,  $0 + j 2.4$ . The reactance is therefore inductive, equal to  $600 \times 2.4 = 1440 \Omega$ . The same line when open-circuited (termination impedance =  $\infty$ , the point at the right of the chart) is represented by  $\ell 2$  in Fig 11. At B the normalized line-input impedance may be read as  $0 - j 0.41$ ; the reactance in this case is capacitive,  $600 \times 0.41 = 246 \Omega$ . (Line losses are disregarded in these examples.) From Fig 11 it is easy to visualize that if  $\ell 1$  were to be extended by  $1/4 \lambda$ , the total length represented by  $\ell 3$ , the line-input impedance would be identical to that obtained in the case represented by  $\ell 2$  alone. In the case of  $\ell 2$ , the line is open-

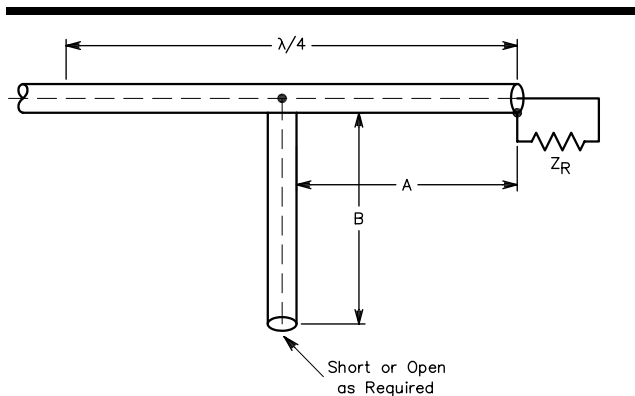


Fig 12—The method of stub matching a mismatched load on coaxial lines.

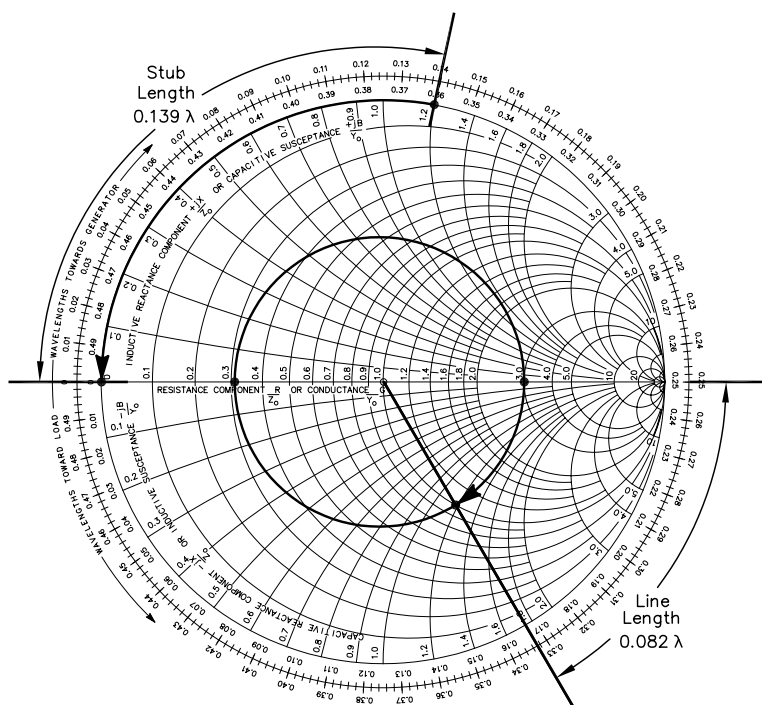


Fig 13—Smith Chart method of determining the dimensions for stub matching.

circuited at the far end, but in the case of  $\ell 3$  the line is terminated in a short. The added section of line for  $\ell 3$  provides the “transformer action” for which the  $1/4\text{-}\lambda$  line is noted.

The equivalent inductance and capacitance as determined above can be found by substituting these values in the equations relating inductance and capacitance to reactance, or by using the various charts and calculators available. The frequency corresponding to the line length in degrees must be used, of course. In this example, if the frequency is 14 MHz the equivalent inductance and capacitance in the two cases are  $16.4 \mu\text{H}$  and  $46.2 \text{ pF}$ , respectively. Note that when the line length is  $45^\circ$  ( $0.125 \lambda$ ), the reactance in either case is numerically equal to the characteristic impedance of the line. In using the Smith Chart it should be kept in mind that the electrical length of a line section depends on the frequency and velocity of propagation, as well as on the actual physical length.

At lengths of line that are exact multiples of  $1/4 \lambda$ , such lines have the properties of resonant circuits. At lengths where the input reactance passes through zero at the left of the Smith Chart, the line acts as a series-resonant circuit. At lengths for which the reactances theoretically pass from “positive” to “negative” infinity at the right of the Smith Chart, the line simulates a parallel-resonant circuit.

### Designing Stub Matches with the Smith Chart

The design of stub matches is covered in detail in Chapter 26. Equations are presented there to calculate the electrical lengths of the main line and the stub, based on a purely resistive load and on the stub being the same type of line as the main line. The Smith Chart may also be used to determine these lengths, without the requirements that the load be purely resistive and that the line types be identical.

Fig 12 shows the stub matching arrangement in coaxial line. As an example, suppose that the load is an antenna, a close-spaced array fed with a  $52\text{-}\Omega$  line. Further suppose that the SWR has been measured as 3.1:1. From this information, a constant SWR circle may be drawn on the Smith Chart. Its radius is such that it intersects the right portion of the resistance axis at the SWR value, 3.1, as shown at point B in Fig 13.

Since the stub of Fig 12 is connected in parallel with the transmission line, determining the design of the matching arrangement is simplified if Smith Chart values are dealt with as admittances, rather than impedances. (An admittance is simply the reciprocal of the associated impedance. Plotted on the Smith Chart, the two associated points are on the same SWR circle, but diametrically opposite each other.) Using admittances leaves less chance for errors

in making calculations, by eliminating the need for making series-equivalent to parallel-equivalent circuit conversions and back, or else for using complicated equations for determining the resultant value of two complex impedances connected in parallel.

A complex impedance,  $Z$ , is equal to  $R + jX$ . The equivalent admittance,  $Y$ , is equal to  $G - jB$ , where  $G$  is the conductive component and  $B$  the susceptance. (Inductance is taken as negative susceptance, and capacitance as positive.) Conductance and susceptance values are plotted and handled on the Smith Chart in the same manner as resistance and reactance.

Assuming that the close-spaced array of our example has been resonated at the operating frequency, it will present a purely resistive termination for the load end of the 52- $\Omega$  line. It is known that the impedance of the antenna equals  $Z_0/\text{SWR} = 52/3.1 = 16.8 \Omega$ . (We can logically discard the possibility that the antenna impedance is  $\text{SWR} \times Z_0$ , or 0.06  $\Omega$ .) If this 16.8- $\Omega$  value were to be plotted as an impedance on the Smith Chart, it would first be normalized ( $16.8/52 = 0.32$ ) and then plotted as  $0.32 + j0$ . Although not necessary for the solution of this example, this value is plotted at point A in Fig 13. What is necessary is a plot of the admittance for the antenna as a load. This is the reciprocal of the impedance;  $1/16.8 \Omega$  equals 0.060 siemen. To plot this point it is first normalized by multiplying the conductance and susceptance values by the  $Z_0$  of the line. Thus,  $(0.060 + j0) \times 52 = 3.1 + j0$ . This admittance value is shown plotted at point B in Fig 13. It may be seen that points A and B are diametrically opposite each other on the chart. Actually, for the solution of this example, it wasn't necessary to compute the values for either point A or point B as in the above paragraph, for they were both determined from the known SWR value of 3.1. As may be seen in Fig 13, the points are located on the constant SWR circle which was already drawn, at the two places where it intersects the resistance axis. The plotted value for point A, 0.32, is simply the reciprocal of the value for point B, 3.1. However, an understanding of the relationship between impedance and admittance is easier to gain with simple examples such as this.

In stub matching, the stub is to be connected at a point in the line where the conductive component equals the  $Z_0$  of the line. Point B represents the admittance of the load, which is the antenna. Various admittances will be encountered along the line, when moving in a direction indicated by the TOWARD GENERATOR wavelengths scale, but all admittance plots must fall on the constant SWR circle. Moving clockwise around the SWR circle from point B, it is seen that the line input conductance will be 1.0 (normalized  $Z_0$  of the line) at point C,  $0.082 \lambda$  toward the transmitter from the antenna. Thus, the stub should be connected at this location on the line.

The normalized admittance at point C, the point representing the location of the stub, is  $1 - j1.2$  siemens, having an inductive susceptance component. A capacitive susceptance having a normalized value of  $+j1.2$  siemens is required across the line at the point of stub connection, to cancel the inductance. This capacitance is to be obtained from the stub section itself; the problem now is to determine its type of

termination (open or shorted), and how long the stub should be. This is done by first plotting the susceptance required for cancellation,  $0 + j1.2$ , on the chart (point D in Fig 13). This point represents the input admittance as seen looking into the stub. The "load" or termination for the stub section is found by moving in the TOWARD LOAD direction around the chart, and will appear at the closest point on the resistance/conductance axis, either at the left or the right of the chart. Moving counterclockwise from point D, this is located at E, at the left of the chart,  $0.139 \lambda$  away. From this we know the required stub length. The "load" at the far end of the stub, as represented on the Smith Chart, has a normalized admittance of  $0 + j0$  siemen, which is equivalent to an open circuit.

When the stub, having an input admittance of  $0 + j1.2$  siemens, is connected in parallel with the line at a point  $0.082 \lambda$  from the load, where the line input admittance is  $1.0 - j1.2$ , the resultant admittance is the sum of the individual admittances. The conductance components are added directly, as are the susceptance components. In this case,  $1.0 - j1.2 + j1.2 = 1.0 + j0$  siemen. Thus, the line from the point of stub connection to the transmitter will be terminated in a load which offers a perfect match. When determining the physical line lengths for stub matching, it is important to remember that the velocity factor for the type of line in use must be considered.

## MATCHING WITH LUMPED CONSTANTS

It was pointed out earlier that the purpose of a matching stub is to cancel the reactive component of line impedance at the point of connection. In other words, the stub is simply a reactance of the proper kind and value shunted across the line. It does not matter what physical shape this reactance takes. It can be a section of transmission line or a "lumped" inductance or capacitance, as desired. In the above example with the Smith Chart solution, a capacitive reactance was required. A capacitor having the same value of reactance can be used just as well. There are cases where, from an installation standpoint, it may be considerably more convenient to connect a capacitor in place of a stub. This is particularly true when open-wire feeders are used. If a variable capacitor is used, it becomes possible to adjust the capacitance to the exact value required.

The proper value of reactance may be determined from Smith Chart information. In the previous example, the required susceptance, normalized, was  $+j1.2$  siemens. This is converted into actual siemens by dividing by the line  $Z_0$ ;  $1.2/52 = 0.023$  siemen, capacitance. The required capacitive reactance is the reciprocal of this latter value,  $1/0.023 = 43.5 \Omega$ . If the frequency is 14.2 MHz, for instance, 43.5  $\Omega$  corresponds to a capacitance of 258 pF. A 325-pF variable capacitor connected across the line  $0.082 \lambda$  from the antenna terminals would provide ample adjustment range. The RMS voltage across the capacitor is

$$E = \sqrt{P \times Z_0}$$

For 500 W, for example,  $E =$  the square root of  $500 \times 52 = 161$  V. The peak voltage is 1.41 times the RMS value, or 227 V.

# The Series-Section Transformer

The series-section transformer can be designed graphically with the aid of a Smith Chart. This information is based on a *QST* article by Frank A. Regier, OD5CG. Using the Smith Chart to design a series-section match requires the use of the chart in its less familiar off-center mode. This mode is described in the next two paragraphs.

**Fig 14** shows the Smith Chart used in its familiar centered mode, with all impedances normalized to that of the transmission line, in this case  $75\ \Omega$ , and all constant SWR circles concentric with the normalized value  $r = 1$  at the chart center. An actual impedance is recovered by multiplying a chart reading by the normalizing impedance of  $75\ \Omega$ . If the actual (unnormalized) impedances represented by a constant SWR circle in Fig 14 are instead divided by a normalizing impedance of  $300\ \Omega$ , a different picture results. A Smith Chart shows all possible impedances, and so a closed path such as a constant SWR circle in Fig 14 must again be represented by a closed path. In fact, it can be shown that the path remains a circle, but that the constant SWR circles are no longer concentric. **Fig 15** shows the circles that result when the impedances along a mismatched  $75\text{-}\Omega$  line are normalized by dividing by  $300\ \Omega$  instead of  $75$ . The constant SWR circles still surround the point corresponding to the characteristic impedance of the line ( $r = 0.25$ ) but are no longer concentric with it. Note that the normalized impedances read from corresponding points on Figs 14 and 15 are different but that the actual, unnormalized, impedances are exactly the same.

## An Example

Now turn to the example shown in **Fig 16**. A complex load of  $Z_L = 600 + j\ 900\ \Omega$  is to be fed with  $300\text{-}\Omega$  line, and a  $75\text{-}\Omega$  series section is to be used. These characteristic impedances agree with those used in Fig 15, and thus Fig 15 can be used to find the impedance variation along the  $75\text{-}\Omega$  series section. In particular, the constant SWR circle which passes through the Fig 15 chart center,  $\text{SWR} = 4$  in this case, passes through all the impedances (normalized to  $300\ \Omega$ ) which the  $75\text{-}\Omega$  series section is able to match to the  $300\text{-}\Omega$  main line. The length  $\ell_1$  of  $300\text{-}\Omega$  line has the job of transforming the load impedance to some impedance on this matching circle.

**Fig 17** shows the whole process more clearly, with all impedances normalized to  $300\ \Omega$ . Here the normalized load impedance  $Z_L = 2 + j\ 3$  is shown at R, and the matching circle appears centered on the resistance axis and passing through the points  $r = 1$  and  $r = n^2 = (75/300)^2 = 0.0625$ . A constant SWR circle is drawn from R to an intersection with the matching circle at Q or Q' and the corresponding length  $\ell_1$  (or  $\ell_1'$ ) can be read directly from the Smith Chart. The clockwise distance around the matching circle represents the length of the matching line, from either Q' to P or from Q to P. Because in this example the distance QP is the shorter of the two for the matching section, we choose the length  $\ell_1$  as shown. By using values from the TOWARD GENERATOR scale,

this length is found as  $0.045 - 0.213$ , and adding  $0.5$  to obtain a positive result yields a value of  $0.332\ \lambda$ .

Although the impedance locus from Q to P is shown in Fig 17, the length  $\ell_2$  cannot be determined directly from this chart. This is because the matching circle is not concentric with the chart center, so the wavelength scales do not apply to this circle. This problem is overcome by forming **Fig 18**, which is the same as Fig 17 except that all normalized impedances have been divided by  $n = 0.25$ , resulting in a Smith Chart normalized to  $75\ \Omega$  instead of  $300$ . The matching circle and the chart center are now concentric, and the series-section length  $\ell_2$ , the distance between Q and P, can be taken directly from the chart. By again using the TOWARD GENERATOR scale, this length is found as  $0.250 - 0.148 = 0.102\ \lambda$ .

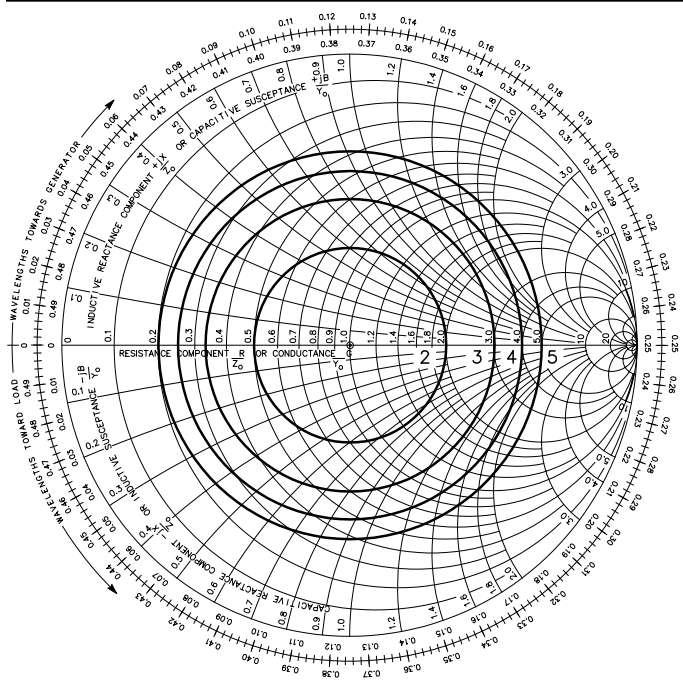
In fact it is not necessary to construct the entire impedance locus shown in Fig 18. It is sufficient to plot  $Z_Q/n$  ( $Z_Q$  is read from Fig 17) and  $Z_p/n = 1/n$ , connect them by a circular arc centered on the chart center, and to determine the arc length  $\ell_2$  from the Smith Chart.

## Procedure Summary

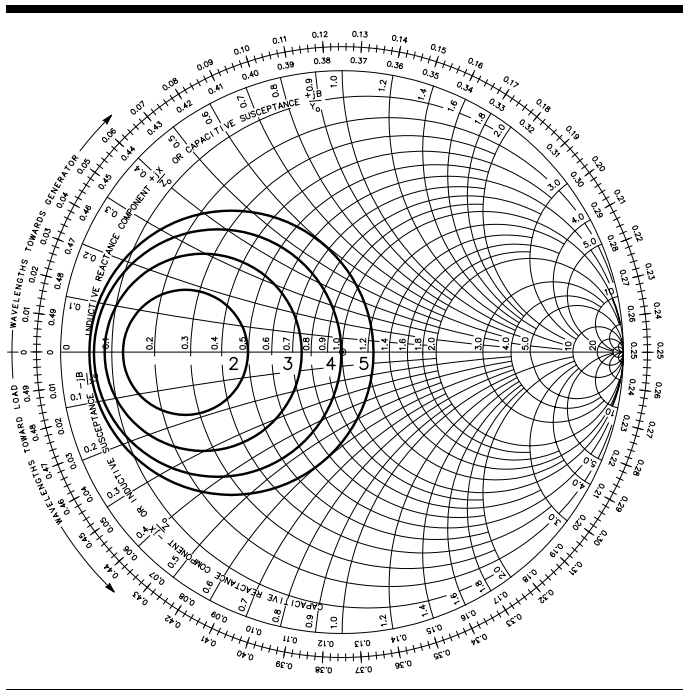
The steps necessary to design a series-section transformer by means of the Smith Chart can now be listed:

- 1) Normalize all impedances by dividing by the characteristic impedance of the main line.
- 2) On a Smith Chart, plot the normalized load impedance  $Z_L$  at R and construct the matching circle so that its center is on the resistance axis and it passes through the points  $r = 1$  and  $r = n^2$ .
- 3) Construct a constant SWR circle centered on the chart center through point R. This circle should intersect the matching circle at two points. One of these points, normally the one resulting in the shorter clockwise distance along the matching circle to the chart center, is chosen as point Q, and the clockwise distance from R to Q is read from the chart and taken to be  $\ell_1$ .
- 4) Read the impedance  $Z_Q$  from the chart, calculate  $Z_Q/n$  and plot it as point Q on a second Smith Chart. Also plot  $r = 1/n$  as point P.
- 5) On this second chart construct a circular arc, centered on the chart center, clockwise from Q to P. The length of this arc, read from the chart, represents  $\ell_2$ . The design of the transformer is now complete, and the necessary physical line lengths may be determined.

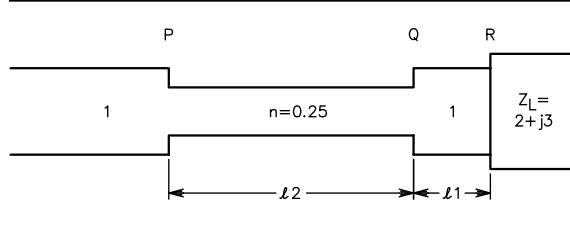
The Smith Chart construction shows that two design solutions are usually possible, corresponding to the two intersections of the constant SWR circle (for the load) and the matching circle. These two values correspond to positive and negative values of the square-root radical in the equation for a mathematical solution of the problem. It may happen, however, that the load circle misses the matching circle completely, in which case no solution is possible. The cure is to enlarge the matching circle by choosing a series section whose impedance departs more from that of the main line.



**Fig 14—Constant SWR circles for SWR = 2, 3, 4 and 5, showing impedance variation along 75-Ω line, normalized to 75 Ω. The actual impedance is obtained by multiplying the chart reading by 75 Ω.**



**Fig 15—Paths of constant SWR for SWR = 2, 3, 4 and 5, showing impedance variation along 75-Ω line, normalized to 300 Ω. Normalized impedances differ from those in Fig 14, but actual impedances are obtained by multiplying chart readings by 300 Ω and are the same as those corresponding in Fig 14. Paths remain circles but are no longer concentric. One, the matching circle, SWR = 4 in this case, passes through the chart center and is thus the locus of all impedances which can be matched to a 300-Ω line.**



**Fig 16—Example for solution by Smith Chart. All impedances are normalized to 300 Ω.**

A final possibility is that, rather than intersecting the matching circle, the load circle is tangent to it. There is then but one solution—that of the  $1/4\lambda$  transformer.

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Source material and more extended discussion of topics covered in this chapter can be found in the references given below and in the textbooks listed at the end of the Antenna Fundamentals chapter of *The ARRL Antenna Book*.

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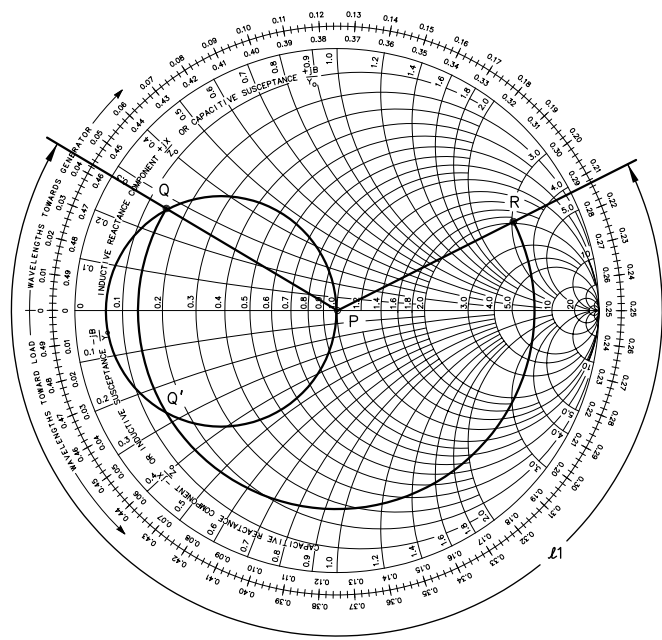


Fig 17—Smith Chart representation of the example shown in Fig 16. The impedance locus always takes a clockwise direction from the load to the generator. This path is first along the constant SWR circle from the load at R to an intersection with the matching circle at Q or Q', and then along the matching circle to the chart center at P. Length  $\ell_1$  can be determined directly from the chart, and in this example is  $0.332 \lambda$ .

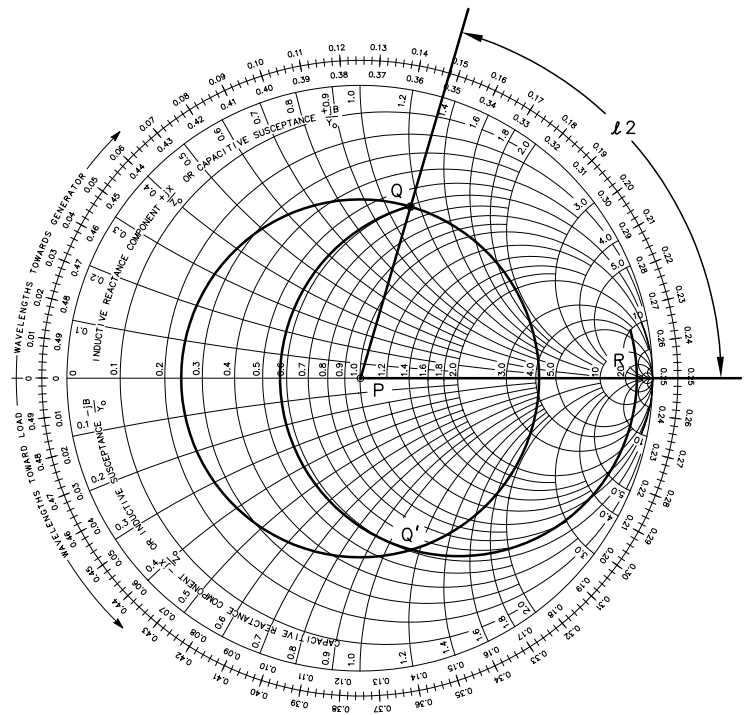


Fig 18—The same impedance locus as shown in Fig 17 except normalized to  $75 \Omega$  instead of  $300$ . The matching circle is now concentric with the chart center, and  $\ell_2$  can be determined directly from the chart,  $0.102 \lambda$  in this case.

