Letters to the Editor

An Automated Method for Measuring Quartz Crystals (Nov/Dec 2013)

Hi Larry,

Readers may like to note that in the article "An Automated Method for Measuring Quartz Crystals" by Richard J. Harris, G3OTK, *QEX* Nov/Dec 2013 pp 3-8, an exact formula can in fact be written for C_h. The formidable looking expressions in the text hide the fact that the algebra contains numerous cancellations. Using the valid approximation:

$\left(\frac{f_n}{f_0}\right)^2 - 1 \approx 2\left(\frac{f_n}{f_0} - 1\right)$ we can derive:

$$C_h = \frac{C_2^{\parallel}\left(f_2 - f_0\right)\left[C_{osc} - \left(\frac{f_1}{f_0}\right)^2 C_1^{\parallel}\right] - C_1^{\parallel}\left(f_1 - f_0\right)\left[C_{osc} - \left(\frac{f_2}{f_0}\right)^2 C_2^{\parallel}\right]}{\left(f_1 - f_0\right)\left[C_{osc} - \left(\frac{f_2}{f_0}\right)^2 C_2^{\parallel}\right] - \left(f_2 - f_0\right)\left[C_{osc} - \left(\frac{f_1}{f_0}\right)^2 C_1^{\parallel}\right]} \quad \text{[Eq 1]}$$

where:

$$C_n^{\parallel} = \frac{C_n C_{osc}}{C_n + C_{osc}} \quad \text{[Eq 2]}$$

(n = 1, 2) are the series capacitances as given in the text. Similar to Equation 12 in the text, which can be rewritten as Equation 3:

$$C_m = 2\left(\frac{f_0 - f_s}{f_s}\right)\left(C_h + C_{osc}\right) \quad \text{[Eq 3]}$$

we can also easily show that:

$$C_m = 2\left(\frac{f_n - f_s}{f_s}\right)\left(C_h + C_n^{\parallel}\right) \quad \text{[Eq 4]}$$

Note that in the limit C_{osc} going to infinity then $C_n^{\parallel} \approx C_n$ and we recover the familiar Equation 2 from the August 2007 *QST* Technical Correspondence letter by Jack Smith, K8ZOA, "Measuring Motional Parameters of a Quartz Crystal," but it is not identical to it:[1]

$$C_m = 2\left(\frac{f_n - f_s}{f_s}\right)\left(C_h + C_n\right) \quad \text{[Eq 5]}$$

From the above results we can also obtain Equation 6:

$$C_m \approx 2\left(\frac{f_n - f_0}{f_0}\right)\left[\frac{\left(C_h + C_n^{\parallel}\right)\left(C_h + C_{osc}\right)}{\left(C_{osc} - C_n^{\parallel}\right)}\right] \quad \text{[Eq 6]}$$

This approximation is good for $C_m << (C_h + C_{osc})$, which in the limit of C_{osc} going to infinity is identical with Equation 2 from Jack Smith's Technical Correspondence:

$$C_m \approx 2\left(\frac{f_n - f_0}{f_0}\right)\left(C_h + C_n\right) \quad \text{[Eq 7]}$$

Using values from Figure 1 of Jack Smith's Technical Correspondence, we find a discrepancy of just under 7% too low from the formula of Equation 6, in agreement with Jack's remarks of a – 5% further discrepancy in his Equation 2 from vector network analyzer measurements. Unfortunately no simple fudge factor formula as he suggested for a capacitance can be added to C_n in Equation 7 to obtain agreement with Equation 6. Using my Equation 6, we can also derive a simpler formula for C_h than my Equation 1:

$$C_h \approx \frac{C_2^{\parallel}\left(f_2 - f_0\right)\left[C_{osc} - C_1^{\parallel}\right] - C_1^{\parallel}\left(f_1 - f_0\right)\left[C_{osc} - C_2^{\parallel}\right]}{\left(f_1 - f_0\right)\left[C_{osc} - C_2^{\parallel}\right] - \left(f_2 - f_0\right)\left[C_{osc} - C_1^{\parallel}\right]} \quad \text{[Eq 8]}$$

In view of the simpler formulas given in Equations 6 and 8, which can easily be coded into a program or spreadsheet, they can be used for finding C_h and C_m with an accuracy of at least 2%, without having to perform a numerical solution.

Note that all the formulas are subject to 1 / Q^2 corrections (See the *QEX* Letters to the Editor by Alan, Bloom N1AL.[2]), which are insignificant for high *Q* HF crystals but may be important for VHF and higher frequency overtone crystals. This is a subject recently investigated further by Jim Koehler, VE5FP.[3]

— 73, Tuck Choy, MØTCC; **m0tcc@arrl.net**

Hi Larry,

I must congratulate Tuck, MØTCC, for deriving an equation for the holder capacitance of crystals. I had tried to derive such a formula myself but gave up when I realized that I could achieve the same result using a numerical method. I have used Tuck's formula (Equation 1) with data that I measured and I can confirm that it yields the same values for holder capacitance within ± 0.1 pF. Such small differences may well be due to measurement uncertainty.

In my article I mentioned that I had not read of a method of automatically measuring the motional parameters and holder capacitance of crystals. I have subsequently learned of another method that was described by Jim Koehler, VE5FP, in an article entitled "Some Thoughts on Crystal Parameter Measurement," and which was published in the July/August 2008 edition of *QEX*, as noted by Tuck. (See Note 3.)

—73, Richard Harris, G3OTK, 10 South St, South Petherton, Somerset TA13 5AD, United Kingdom; **r.j.harris.g3otk@gmail.com**

Notes

[1]Jack Smith, K8ZOA, "Measuring Motional Parameters of a Quartz Crystal (Technical Correspondence)," *QST* Aug 2007, pp 74-75. A straightforward derivation in a Colpitts oscillator configuration is also given by Andrew Smith, G4OEP on his website: **http://g4oep.atspace.com/crystal%20parameters/deriving_g3uur.htm** but the Colpitts oscillator configuration with C_{osc} assumed infinite is in fact not necessary for its derivation.

[2]Alan Bloom, N1AL, "Letters to the Editor," *QEX* Sep/Oct 2008, pp 41-42 and *QEX* Jan/Feb 2010, pp 37-38.

[3]Jim Koehler, VE5FP, "The Shunt Method for Crystal Parameter Measurement," *QEX* Jul/Aug 2010, pp 16-25. Jim performed a quite thorough analysis of the discrepancies in various formulas for overtone crystals.